# CSE 311: Foundations of Computing I

## Logical Equivalences Reference Sheet

### Identity

$$p \land \mathsf{T} \equiv p$$
$$p \lor \mathsf{F} \equiv p$$

### Domination

$$p \lor \mathsf{T} \equiv \mathsf{T}$$
$$p \land \mathsf{F} \equiv \mathsf{F}$$

### Idempotency

$$p \lor p \equiv p$$
$$p \land p \equiv p$$

### Commutativity

$$p \lor q \equiv q \lor p$$
$$p \land q \equiv q \land p$$

### Associativity

$$(p \lor q) \lor r \equiv p \lor (q \lor r)$$
$$(p \land q) \land r \equiv p \land (q \land r)$$

### Distributivity

$$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$$
$$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$$

### Absorption

$$p \lor (p \land q) \equiv p$$
$$p \land (p \lor q) \equiv p$$

### Negation

$$p \lor \neg p \equiv \mathsf{T}$$
$$p \land \neg p \equiv \mathsf{F}$$

### DeMorgan's Laws

$$\neg(p \lor q) \equiv \neg p \land \neg q$$
$$\neg(p \land q) \equiv \neg p \lor \neg q$$

### Double Negation

$$\neg\neg p \equiv p$$

### Law of Implication

$$p \to q \equiv \neg p \lor q$$

### Contrapositive

$$p \to q \equiv \neg q \to \neg p$$

## Boolean Algebra Axioms (and Some Theorems)

# Axioms

### Closure

$a + b$ is in $\mathbb{B}$

$a \bullet b$ is in $\mathbb{B}$

### Commutativity

$$a + b = b + a$$
$$a \bullet b = b \bullet a$$

### Associativity

$$a + (b + c) = (a + b) + c$$
$$a \bullet (b \bullet c) = (a \bullet b) \bullet c$$

### Identity

$$a + 0 = a$$
$$a \bullet 1 = a$$

### Distributivity

$$a + (b \bullet c) = (a + b) \bullet (a + c)$$
$$a \bullet (b + c) = (a \bullet b) + (a \bullet c)$$

### Complementarity

$$a + a' = 1$$
$$a \bullet a' = 0$$

# Theorems

### Null

$$X + 1 = 1$$
$$X \bullet 0 = 0$$

### Idempotency

$$X + X = X$$
$$X \bullet X = X$$

### Involution

$$(X')' = X$$

### Uniting

$$X \bullet Y + X \bullet Y' = X$$
$$(X + Y) \bullet (X + Y') = X$$

### Absorbtion

$$X + X \bullet Y = X$$
$$(X + Y') \bullet Y = X \bullet Y$$
$$X \bullet (X + Y) = X$$
$$(X \bullet Y') + Y = X + Y$$

### DeMorgan

$$(X + Y + \cdots)' = X' \bullet Y' \bullet \cdots$$
$$(X \bullet Y \bullet \cdots)' = X' + Y' + \cdots$$

### Consensus

$$(X \bullet Y) + (Y \bullet Z) + (X' \bullet Z) = X \bullet Y + X' \bullet Z$$
$$(X + Y) \bullet (Y + Z) \bullet (X' + Z) = (X + Y) \bullet (X' + Z)$$

### Factoring

$$(X + Y) \bullet (X' + Z) = X \bullet Z + X' \bullet Y$$
$$X \bullet Y + X' \bullet Z = (X + Z) \bullet (X' + Y)$$

## Axioms & Inference Rules

**Excluded Middle**

$$\frac{}{\therefore\ A \vee \neg A}$$

**Direct Proof**

$$\frac{A \Rightarrow B}{\therefore\ A \to B}$$

**Modus Ponens**

$$\frac{A \qquad A \to B}{\therefore\ B}$$

**Intro $\wedge$**

$$\frac{A \qquad B}{\therefore\ A \wedge B}$$

**Elim $\wedge$**

$$\frac{A \wedge B}{\therefore\ A \qquad B}$$

**Intro $\vee$**

$$\frac{A}{\therefore\ A \vee B \qquad B \vee A}$$

**Elim $\vee$**

$$\frac{A \vee B \qquad \neg A}{\therefore\ B}$$

**Intro $\forall$**

$$\frac{\text{Let } a \text{ be an arbitrary} \ldots}{\therefore\ \forall x\ P(x)}$$

**Elim $\forall$**

$$\frac{\forall x\ P(x)}{\therefore\ P(a) \text{ for any } a}$$

**Intro $\exists$**

$$\frac{P(c) \text{ for some } c}{\therefore\ \exists x\ P(x)}$$

**Elim $\exists$**

$$\frac{\exists x\ P(x)}{\therefore\ P(c) \text{ for some } \textit{special } c}$$

# CSE 311: Foundations of Computing I

## Set Definitions

### Common Sets

- $\mathbb{N} = \{0, 1, 2, \ldots\}$ is the set of *Natural Numbers*.
- $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ is the set of *Integers*.
- $\mathbb{Q} = \left\{\frac{p}{q} \ : \ p, q \in \mathbb{Z} \wedge q \neq 0\right\}$ is the set of *Rational Numbers*.
- $\mathbb{R}$ is the set of *Real Numbers*.

### Containment, Equality, and Subsets

Let $A, B$ be sets. Then:

- $x \in A$ ("$x$ is an *element* of $A$") means that $x$ is an element of $A$.
- $x \notin A$ ("$x$ is *not* an *element* of $A$") means that $x$ is *not* an element of $A$.
- $A \subseteq B$ ("$A$ is a *subset* of $B$") means that all the elements of $A$ are also in $B$.
- $A \supseteq B$ ("$A$ is a *superset* of $B$") means that all the elements of $B$ are also in $A$.
- $(A = B) \equiv (A \subseteq B) \wedge (B \subseteq A) \equiv \forall x\, (x \in A \leftrightarrow x \in B)$

### Set Operations

Let $A, B$ be sets. Then:

- $A \cup B$ is the *union* of $A$ and $B$. $A \cup B = \{x \ : \ x \in A \vee x \in B\}$.
- $A \cap B$ is the *intersection* of $A$ and $B$. $A \cap B = \{x \ : \ x \in A \wedge x \in B\}$.
- $A \setminus B$ is the *difference* of $A$ and $B$. $A \setminus B = \{x \ : \ x \in A \wedge x \notin B\}$.
- $A \oplus B$ is the *symmetric difference* of $A$ and $B$. $A \oplus B = \{x \ : \ x \in A \oplus x \in B\}$.
- $\overline{A}$ is the *complement* of $A$. If we restrict ourselves to a "universal set", $\mathcal{U}$, (a set of all possible things we're discussing), then $\overline{A} = \{x \in \mathcal{U} \ : \ x \notin A\}$.

### Set Constructions

Let $A, B, C, D$ be sets. Then:

- $S = \{x \ : \ \mathsf{P}(x)\}$ is *set comprehension notation* which means $S$ is a set that contains all objects $x$ with property $P$.
- $A \times B$ is the *cartesian product* of $A$ and $B$. $A \times B = \{(a, b) \ : \ a \in A,\ b \in B\}$.
- $[n]$ ("*brackets* $n$") is the set of natural numbers from 1 to $n$. $[n] = \{x \in \mathbb{N} \ : \ 1 \leq x \leq n\}$.
- $\mathcal{P}(A)$ is the *power set* of $A$. $\mathcal{P}(A) = \{S \ : \ S \subseteq A\}$.