Lecture 22: DFAs and Finite State Machines with Output
Finite State Machines

- States
- Transitions on input symbols
- Start state and final states
- The “language recognized” by the machine is the set of strings that reach a final state from the start

<table>
<thead>
<tr>
<th>Old State</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_0$</td>
<td>$s_0$</td>
<td>$s_1$</td>
</tr>
<tr>
<td>$s_1$</td>
<td>$s_0$</td>
<td>$s_2$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>$s_0$</td>
<td>$s_3$</td>
</tr>
<tr>
<td>$s_3$</td>
<td>$s_3$</td>
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</tbody>
</table>
Finite State Machines

• Each machine designed for strings over some fixed alphabet $\Sigma$.

• Must have a transition defined from each state for every symbol in $\Sigma$.

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Strings over \{0, 1, 2\}

\(M_1\): Strings with an even number of 2’s

\(M_2\): Strings where the sum of digits mod 3 is 0
Strings over \{0, 1, 2\}

\[ M_1: \text{Strings with an even number of 2's} \]

\[ M_2: \text{Strings where the sum of digits mod 3 is 0} \]
Strings over \{0, 1, 2\}

\(M_1\): Strings with an even number of 2’s

\(M_2\): Strings where the sum of digits mod 3 is 0
What language does this machine recognize?
What language does this machine recognize?

The set of all binary strings with \# of 1’s ≡ \# of 0’s (mod 2) (both are even or both are odd).
Strings over \{0, 1, 2\}

\(M_1\): Strings with an even number of 2’s

\(M_2\): Strings where the sum of digits mod 3 is 0
Strings over \{0,1,2\} w/ even number of 2’s and mod 3 sum 0
Strings over \{0,1,2\} w/ even number of 2’s and mod 3 sum 0
Strings over \{0,1,2\} w/ even number of 2’s OR mod 3 sum 0?
Strings over \{0,1,2\} w/ even number of 2’s OR mod 3 sum 0
The set of binary strings with a 1 in the 3\textsuperscript{rd} position from the start
The set of binary strings with a 1 in the 3\textsuperscript{rd} position from the start
The set of binary strings with a 1 in the 3rd position from the end
3 bit shift register  "Remember the last three bits"
The set of binary strings with a 1 in the 3\textsuperscript{rd} position from the end
The set of binary strings with a 1 in the 3\textsuperscript{rd} position from the end
The beginning versus the end
Adding Output to Finite State Machines

• So far we have considered finite state machines that just accept/reject strings
  – called “Deterministic Finite Automata” or DFAs

• Now we consider finite state machines that with output
  – These are the kinds used as controllers
Enter 15 cents in dimes or nickels
Press S or B for a candy bar
Basic transitions on N (nickel), D (dime), B (butterfinger), S (snickers)
Adding output to states: \( N \) – Nickel, \( S \) – Snickers, \( B \) – Butterfinger
Adding additional “unexpected” transitions to cover all symbols for each state