1. DFAs, Stage 1

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1, 2, 3\}$.

(a) All binary strings.

Solution:

\[ q_0 \rightarrow 0, 1 \rightarrow q_0, 2, 3 \rightarrow q_1 \]

$q_0$: binary strings

$q_1$: strings that contain a character which is not 0 or 1.

(b) All strings whose digits sum to an even number.

Solution:

\[ q_0 \rightarrow 0, 2 \rightarrow q_0, 1, 3 \rightarrow q_1 \]

(c) All strings whose digits sum to an odd number.

Solution:

\[ q_0 \rightarrow 0, 2 \rightarrow q_0, 1, 3 \rightarrow q_1 \]

2. DFAs, Stage 2

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1\}$.

(a) All strings which do not contain the substring 101.

Solution:

\[ q_0 \rightarrow 0 \rightarrow q_1 \rightarrow 0 \rightarrow q_2 \rightarrow 1 \rightarrow q_3 \]

\[ 0, 1 \rightarrow q_0 \]

\[ 0, 1 \rightarrow q_1 \]
$q_3$: string that contain 101.
$q_2$: strings that don’t contain 101 and end in 10.
$q_1$: strings that don’t contain 101 and end in 1.
$q_0$: $\varepsilon$, 0, strings that don’t contain 101 and end in 00.

(b) All strings containing at least two 0’s and at most one 1.

**Solution:**

(c) All strings containing an even number of 1’s and an odd number of 0’s and not containing the substring 10.

**Solution:**
3. NFAs
(a) What language does the following NFA accept?

\[ q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3 \]

**Solution:**
All strings of only 0’s and 1’s not containing more than one 1.

(b) Create an NFA for the language “all binary strings that have a 1 as one of the last three digits”.

**Solution:**
The following is one such NFA:

4. DFAs & Minimization
(a) Convert the NFA from 3a to a DFA, then minimize it.

**Solution:**

Here is the minimized form:
(b) Minimize the following DFA:

![DFA Diagram]

**Solution:**

**Step 1:** $q_0, q_2$ are final states and the rest are not final. So, we start with the initial partition with the following groups: group 1 is \{q_0, q_2\} and group 2 is \{q_1, q_3, q_4\}.

**Step 2:** $q_1$ is sending $a$ to group 1 while $q_3, q_4$ are sending $a$ to group 2. So, we divide group 2. We get the following groups: group 1 is \{q_0, q_2\}, group 3 is \{q_1\} and group 4 is \{q_3, q_4\}.

**Step 3:** $q_0$ is sending $a$ to group 3 and $q_2$ is sending $a$ to group 4. So, we divide group 1. We will have the following groups: group 3 is \{q_1\}, group 4 is \{q_3, q_4\}, group 5 is \{q_0\} and group 6 is \{q_2\}.

The minimized DFA is the following:

![Minimized DFA Diagram]