1. Induction with Sums
   (a) Prove for all $n \in \mathbb{N}$ that if you have two groups of numbers, $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$, such that $\forall (i \in [n]). a_i \leq b_i$, then it must be that:
   $\sum_{i=1}^{n} a_i \leq \sum_{i=1}^{n} b_i$

   (b) For any $n \in \mathbb{N}$, define $S_n$ to be the sum of the squares of the first $n$ positive integers, or
   $S_n = \sum_{i=1}^{n} i^2.$
   For all $n \in \mathbb{N}$, prove that $S_n = \frac{1}{6}n(n+1)(2n+1)$.

   (c) Define the triangle numbers as $\triangle_n = 1+2+\cdots+n$, where $n \in \mathbb{N}$. We showed in lecture that $\triangle_n = \frac{n(n+1)}{2}$.
   Prove the following equality for all $n \in \mathbb{N}$:
   $\sum_{i=0}^{n} i^3 = \triangle_n^2$

2. Induction
   (a) Prove that $9 \mid n^3 + (n+1)^3 + (n+2)^3$ for all $n > 1$ by induction.

   (b) Prove that $6n + 6 < 2^n$ for all $n \geq 6$.

   (c) Define
   $H_i = 1 + \frac{1}{2} + \cdots + \frac{1}{i}$
   Prove that $H_{2^n} \geq 1 + \frac{n}{2}$ for $n \in \mathbb{N}$. 

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