1. Pattern Matching [Online] (15 points)
Use the method given in class to design a DFA to determine all occurrences of the string 11011011001 in strings over the alphabet \{0, 1\}.

You must submit and check your answers to this question using https://grinch.cs.washington.edu/cse311/fsm.

2. Diagonalization (20 points)
Let $B$ be the set of all infinite binary sequences that are 1 in odd positions, i.e., any string in $B$ is of the form

$$1 \ast 1 \ast 1 \ast 1 \ast \ldots$$

where we can have $0$ or a $1$ instead of each $\ast$. Show that $B$ is uncountable using a proof by diagonalization.

3. Countability (20 points)
Complex numbers can be written as $a + bi$ where $a, b$ are real numbers and $i$ is the square root of $-1$. Show that subset $R$ of complex numbers given by

$$R = \{a + bi : a, b \text{ are rational}\}$$

is countable

4. Irregularity (30 points)
Using the method shown in class prove that that the following languages are not regular.

(a) [15 Points] The set of binary strings of the form \{0^n1^m0^n : m < n\}.

(b) [15 Points] The set of strings $0^n$ where $n$ is a perfect square, i.e., $n = k^2$ for some $k \in \mathbb{N}$.

5. Undecidability (15 points)
Consider the set

$$\text{Prime} = \{(\text{CODE}(P), x) : P \text{ reads } x \text{ and halts if and only if } x \text{ is a prime}\}$$

Show that $\text{Prime}$ is undecidable using the fact that the Halting Problem is undecidable.