Final Exam Practice Questions

NFA Construction
Draw the state diagram of an NFA $\mathcal{M}$ that recognizes the language $a^*b(b \cup ab)^*a$ over $\Sigma = \{a, b\}$. Try to avoid unnecessary states.

Powerset Construction
Build a DFA equivalent to the following NFA using the powerset construction. You only need to show states that are reachable from the start state of your DFA (but do not simplify further).

DFA, Regexp, CFG
For each of the following languages, construct a DFA, Regular Expression, and CFG for it.
(a) $A = \{w \in \{0, 1\}^* : \text{the number of 0's minus the number of 1's in } w \text{ is divisible by 3}\}$.
(b) $B = \{w \in \{a, b\}^* : \text{every a has two b's immediately to its right}\}$.

Context-Free & Irregular
Consider the language $C = \{a^n ba^m ba^{m+n} : n, m \geq 1\}$.
(a) Show that $C$ is context-free.
(b) Show that $C$ is not regular.

Recursive Definitions & Strong Induction
(a) In the land of Garbanzo, the unit of currency is the bean. They only have two coins, one worth 2 beans and the other worth 5 beans.
   (a) Give a recursive definition of the set of positive integers $S$ such that $x \in S$ iff one can make up an amount worth $x$ beans using at most one 5-bean coin and any number of 2-bean coins.
   (b) Prove by strong induction that if $n \geq 4$, then $n \in S$.
(b) Define $g(n)$ as follows:

\[
g(n + 1) = \begin{cases} 
0 & \text{if } n = 0 \\
\max_{1 \leq k \leq n} g(k) + g(n + 1 - k) + 1 & \text{otherwise}
\end{cases}
\]
Prove by induction that \( g(n) = n - 1 \) for all \( n \geq 1 \).

**Relation Closures**

Let \( R \) be the relation \( \{(1, 2), (3, 4), (1, 3), (2, 1)\} \) defined on the set \( \{1, 2, 3, 4, 5\} \).

(a) Draw the graph of \( R \).

(b) Draw the graph of the \( R^2 \).

(c) Draw the graph of the reflexive-transitive closure of \( R \).

**Relations Proofs**

Suppose \( R_1 \) and \( R_2 \) are reflexive relations on a set \( A \). Is the relation \( R_1 \cup R_2 \) necessarily a reflexive relation? Justify your answer.

**True or False**

For each of the following answer True or False and give a short (1-2 sentence) explanation of your answer.

(a) \( \text{True or False} \) The set \( \{\text{CODE}(R), x\} : R \text{ halts when given } x \} \) is decidable.

(b) \( \text{True or False} \) The set \( \{\text{CODE}(Q) : Q \text{ reads input}\} \) is decidable.

**DFA Minimization**

Minimize the following DFA using the algorithm we discussed in lecture:

![DFA Diagram](image-url)