CSE 311: Foundations of Computing I

Set Definitions

Common Sets
- \( \mathbb{N} = \{0, 1, 2, \ldots \} \) is the set of Natural Numbers.
- \( \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots \} \) is the set of Integers.
- \( \mathbb{Q} = \left\{ \frac{p}{q} : p, q \in \mathbb{Z} \land q \neq 0 \right\} \) is the set of Rational Numbers.
- \( \mathbb{R} \) is the set of Real Numbers.

Containment, Equality, and Subsets
Let \( A, B \) be sets. Then:
- \( x \in A \) ("\( x \) is an element of \( A \)"") means that \( x \) is an element of \( A \).
- \( x \notin A \) ("\( x \) is not an element of \( A \)"") means that \( x \) is not an element of \( A \).
- \( A \subseteq B \) ("\( A \) is a subset of \( B \)"") means that all the elements of \( A \) are also in \( B \).
- \( A \supseteq B \) ("\( A \) is a superset of \( B \)"") means that all the elements of \( B \) are also in \( A \).
- \( (A = B) \equiv (A \subseteq B) \land (B \subseteq A) \equiv \forall x (x \in A \leftrightarrow x \in B) \)

Set Operations
Let \( A, B \) be sets. Then:
- \( A \cup B \) is the union of \( A \) and \( B \). \( A \cup B = \{x : x \in A \lor x \in B\} \).
- \( A \cap B \) is the intersection of \( A \) and \( B \). \( A \cap B = \{x : x \in A \land x \in B\} \).
- \( A \setminus B \) is the difference of \( A \) and \( B \). \( A \setminus B = \{x : x \in A \land x \notin B\} \).
- \( A \oplus B \) is the symmetric difference of \( A \) and \( B \). \( A \oplus B = \{x : x \in A \oplus x \in B\} \).
- \( \overline{A} \) is the complement of \( A \). If we restrict ourselves to a "universal set", \( \mathcal{U} \), (a set of all possible things we’re discussing), then \( \overline{A} = \{x \in \mathcal{U} : x \notin A\} = \{x \in \mathcal{U} : \neg(x \in A)\} \).

Set Constructions
Let \( A, B, C, D \) be sets and \( P \) be a predicate. Then:
- \( S = \{x : P(x)\} \) is notation which means that \( S \) is a set that contains all objects \( x \) (in the domain of \( P \)) with property \( P \).
- \( A \times B \) is the cartesian product of \( A \) and \( B \). \( A \times B = \{(a, b) : a \in A, b \in B\} \).
- \( [n] \) ("brackets \( n \)") is the set of natural numbers from 1 to \( n \). \( [n] = \{x \in \mathbb{N} : 1 \leq x \leq n\} \).
- \( \mathcal{P}(A) \) is the power set of \( A \). \( \mathcal{P}(A) = \{S : S \subseteq A\} \).