1. Pikachu makes a hard choice (12 points)

For each of the first five parts, translate the English statement into logical language. (First introduce variables to represent the atomic propositions, and then use logical connectives to represent the statement.)

(a) Pikachu flies on a private jet only if he gets unlimited peanuts.

(b) Pikachu flies on a private jet whenever he flies on Bieber’s jet or Taylor’s jet.

(c) If Bieber doesn’t talk, Pikachu flies on Bieber’s jet, unless he flies on Taylor’s jet.

(d) Pikachu must wear pants if he flies on Taylor’s jet.

(e) If Pikachu gets unlimited peanuts and flies on Taylor’s jet, then Pikachu wears pants unless (Bieber talks or Pikachu doesn’t fly on a private jet). [I put parentheses just to disambiguate the sentence!]

(f) Let $A, B, C, D, E$ represent the propositions from parts (a), (b), (c), (d), (e), respectively. Is the proposition $(A \land B \land C \land D) \rightarrow E$ a tautology? [Prove or disprove]
2. You’re not odd. You’re just special. (12 points)

(a) Find a compound proposition involving the propositional variables $p, q, r$ that is true precisely when an odd number of the variables $p, q,$ and $r$ are true.

(b) Write down the negation of your proposition from part (a), and then write an equivalent proposition where all negation symbols occur immediately in front of the propositional variables. (For instance, you should not have $\neg(a \land b)$ or $\neg(a \to b)$.)
3. What are the implications? (12 points)

In each of the next four parts, write an equivalent proposition using only the implication $\rightarrow$ and the literals $T$ and $F$ (and the variables $p$ and $q$). You can also use parentheses.

(a) $\neg p$

(b) $p \lor q$

(c) $p \land q$

(d) $p \leftrightarrow q$
4. Contrapositing (10 points)

State in English the converse and contrapositive of each of the following implications.

(a) If a law is broken and the judge finds you guilty, then you will go to jail.
   (Be sure to use de Morgan’s Law to simplify the contrapositive so the statement reads more naturally in English.)

(b) If you always carry an umbrella when it’s raining, then it’s never raining when you get wet.
   (Again, be sure to use de Morgan’s law to simplify the contrapositive. First, you might need to recall that \( p \rightarrow q \equiv \neg p \lor q \).)
5. Vee(∨) v. Wedge(∧) (12 points)

Using only the following gates:

AND Gates, OR Gates, and Inverters (NOT Gates),

Design a circuit with three inputs that computes the function $M(p, q, r)$ where

$M(T, q, r) = q \land r$

$M(F, q, r) = q \lor r$
6. **Extra credit: XNORing**

For two bits $a$ and $b$, we define $\text{XNOR}(a, b) = \neg(a \oplus b)$. Suppose we have two memory registers with the same number of bits. You also have an operator $\text{XNOR}(R_1, R_2)$ which takes the two registers, performs a bitwise XNOR between them, and stores the result in $R_1$.

Show how you can swap the contents of the registers using a sequence of XNORs without temporary memory registers.