1. English Translation [12 points]

Write each of these statements in the form “if $p$, then $q$” in English.

(a) To become a president of the United States, it is necessary to be a natural-born citizen.

(b) Whenever I hike for 10 hours, I have to rest for a day.

(c) To travel to the moon, it is enough to have a billion dollars.

(d) TAs will grade the solutions to the homework when the deadline is passed.
2. Nonequivalent Logical Statements [12 points]

Use truth assignments to show that the two propositions in each part are not logically equivalent:

(a) \( p \lor q \) vs. \( \neg(p \land q) \)

(b) \( (p \oplus q) \lor (p \oplus r) \) vs. \( p \lor q \lor r \)

(c) \( (p \rightarrow q) \rightarrow (q \rightarrow p) \) vs. \( (q \rightarrow p) \rightarrow (p \rightarrow q) \)

(d) \( (((p \rightarrow q) \rightarrow r) \rightarrow s) \rightarrow p \) vs. \( p \rightarrow (q \rightarrow (r \rightarrow (s \rightarrow p))) \)
3. Equivalence of NAND [18 points]

Define the NAND logical connective: \( p\#q \equiv \neg(p \land q) \). In this part, we will see that you can design a digital circuit made up of only the NAND connectives for any logical proposition. For each of the following propositions, write a logically equivalent proposition using only the connective \( \# \) and possibly the literals \( T \) and \( F \).

(a) \( \neg p \)

(b) \( p \land q \)

(c) \( p \lor q \)

(d) \( p \oplus q \)

(e) \( p \rightarrow q \)

(f) \( p \leftrightarrow q \)
4. Simple circuit design [10 points]

Using only the following gates:

- AND Gates
- OR Gates
- Inverters (NOT Gates)

Design a circuit with three inputs that computes the function $M(p, q, r)$ where

- $M(T, q, r) = q \land \neg r$
- $M(F, q, r) = \neg q \lor r$
Extra credit: XNORing

For two bits $a$ and $b$, we define $\text{XNOR}(a, b) = \neg(a \oplus b)$. Suppose we have two memory registers with the same number of bits. You also have an operator $\text{XNOR}(R_1, R_2)$ which takes the two registers, performs a bitwise XNOR between them, and stores the result in $R_1$.

Show how you can swap the contents of the registers using a sequence of XNORs without temporary memory registers.