Structural Induction
(a) Recall the following definitions:

\[ \text{len} (\varepsilon) = 0 \]
\[ \text{len} (wa) = \text{len} (w) + 1, \text{for } w \in \Sigma^*, a \in \Sigma \]

\[ x \cdot \varepsilon = x, \text{for } x \in \Sigma^* \]
\[ x \cdot wa = (x \cdot w)a, \text{for } x \in \Sigma^*, a \in \Sigma \]

Consider the following recursive definition:

\[ \text{stutter} (\varepsilon) = \varepsilon \]
\[ \text{stutter} (wa) = \text{stutter} (w) \cdot aa, \text{for } w \in \Sigma^*, a \in \Sigma \]

Prove that \( \text{len} (\text{stutter} (w)) = 2 \text{len} (w) \) for all \( w \in \Sigma^* \).

Regular Expressions
(a) Write a regular expression that matches base 10 numbers (e.g., there should be no leading zeroes).

(b) Write a regular expression that matches all base-3 numbers that are divisible by 3.

(c) Write a regular expression that matches all binary strings that contain the substring “111”, but not the substring “000”.

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CSE 311: Foundations of Computing I
Section: Structural Induction and Regular Expressions

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1