CSE 311: Foundations of Computing I
Section: Number Theory

GCD
(a) Calculate $\gcd(100, 50)$.
(b) Calculate $\gcd(17, 31)$.
(c) Find the multiplicative inverse of 6 modulo 7.
(d) Does 49 have an multiplicative inverse modulo 7?
(e) Find the multiplicative inverse of 7 modulo 311.
(f) Find the multiplicative inverse of 27 modulo 151.

More Number Theory
(a) Prove that if $n^2 + 1$ is a perfect square, where $n$ is an integer, then $n$ is even.
(b) Prove that if $n$ is a positive integer such that the sum of the divisors of $n$ is $n+1$, then $n$ is prime.

Induction
(a) Prove that if you have two groups of numbers, $a_1, \cdots, a_n$ and $b_1, \cdots, b_n$, such that $\forall (i \in [n]), a_i \leq b_i$, then it must be that:

$$\sum_{i=1}^{n} a_i \leq \sum_{i=1}^{n} b_i$$

(b) For any $n \in \mathbb{N}$, define $S_n$ to be the sum of the squares of the first $n$ positive integers, or

$$S_n = \sum_{i=1}^{n} i^2.$$

For all $n \in \mathbb{N}$, prove that $S_n = \frac{1}{6} n(n+1)(2n+1)$.

(c) Define the triangle numbers as $\triangle_n = 1 + 2 + \cdots + n$, where $n \in \mathbb{N}$. We showed in lecture that $\triangle_n = \frac{n(n+1)}{2}$.

Prove the following equality for all $n \in \mathbb{N}$:

$$\sum_{i=0}^{n} i^3 = \triangle_n^2$$