Binary Addition
Just as a quick recall of binary, do the following operations. Then, convert your answers to base-10.

(a) \((101011)_2 + (1111)_2\)

(b) \((101011)_2 \oplus (1111)_2\)

(c) \((101011)_2 \ast (1111)_2\)

Equivalences
Prove that each of the following pairs of propositional formulae are equivalent using propositional equivalences.

(a) \(p \leftrightarrow q\) \(\equiv (p \land q) \lor (\neg p \land \neg q)\)

(b) \(\neg p \rightarrow (q \rightarrow r)\) \(\equiv q \rightarrow (p \lor r)\)

Tautologies
Prove that each of the following propositional formulae are tautologies by showing they are equivalent to \(T\).

(a) \(((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r)\)

(b) \((p \land q) \lor (p \land r) \rightarrow (q \lor r)\)

(c) \((p \land q) \lor (\neg p \land q) \lor \neg q\)
Non-equivalence

Prove that each of the following pairs of propositional formulae are not equivalent by finding an input they differ on.

(a) \( p \to q \) \( q \to p \)

(b) \((p \to q) \to r\) \( p \to (q \to r)\)

Convert To A Circuit

(a) \( \neg((p \lor q) \land (p \lor r)) \lor (q \lor r) \)

Boolean Algebra

For each of the following parts, write the logical expression using boolean algebra operators. Then, simplify it using axioms and laws of boolean algebra.

(a) \( \neg p \lor (\neg q \lor (p \land q)) \)

(b) \( \neg(p \lor (q \land p)) \)