0. Extended Euclidian Algorithm

Find the multiplicative inverse $y$ of $7 \mod 33$. That is, find $y$ such that $7y \equiv 1 \pmod{33}$. You should use the extended Euclidean Algorithm. Your answer should be in the range $0 \leq y < 29$.

Solution: First, we find the gcd:

\[
gcd(33, 7) = gcd(7, 5) = gcd(5, 2) = gcd(2, 1) = gcd(1, 0) = 1
\]

Next, we re-arrange equations (1) - (3) by solving for the remainder:

\[
1 = 5 - 2 \cdot 2
\]
\[
2 = 7 - 5 \cdot 1
\]
\[
5 = 33 - 7 \cdot 4
\]

Now, we backward substitute into the boxed numbers using the equations:

\[
1 = 5 - 2 \cdot 2
\]
\[
= 5 - (7 - 5 \cdot 1) \cdot 2
\]
\[
= 3 \cdot 5 - 7 \cdot 2
\]
\[
= 3 \cdot (33 - 7 \cdot 4) - 7 \cdot 2
\]
\[
= 33 \cdot 3 + 7 \cdot -14
\]

So, $1 = 33 \cdot 3 + 7 \cdot -14$. Thus, $33 - 14 = 19$ is the multiplicative inverse of $7 \mod 33$. 
