Section Week 9 Worksheet

Solutions
1) NFA to DFA

Convert the following NFA to DFA.

New start state: \{a,c\}

\{a,c\} \rightarrow \{b,d,e\} with a 0
\{a,c\} \rightarrow \text{null} with a 1
1) NFA to DFA

\{b,d,e\} \rightarrow \{e\} with a 0
\{b,d,e\} \rightarrow \{d,e\} with a 1
\{d,e\} \rightarrow \{e\} with a 0
\{d,e\} \rightarrow \{e\} with a 1
\{e\} \rightarrow \text{null} with a 0
\{e\} \rightarrow \text{null} with a 1

Final states (all that contain an e): \{b,d,e\}, \{d,e\}, \{e\}
2) State Minimization

Minimize the following DFA:

Start by dividing into final and non-final states:

G1: \{q_0, q_1, q_2\}  G2: \{q_3\}
2) State Minimization

G1: \{q_0, q_1, q_2\}  G2: \{q_3\}

- \(q_0 \rightarrow G1\) with 0
- \(q_0 \rightarrow G1\) with 1
- \(q_1 \rightarrow G1\) with 0
- \(q_1 \rightarrow G1\) with 1
- \(q_2 \rightarrow G1\) with 0
- \(q_2 \rightarrow G2\) with 1
- \(q_3 \rightarrow G2\) with 0
- \(q_3 \rightarrow G2\) with 1

New groups: \{q_0\}, \{q_1, q_2\}, \{q_3\}
2) State Minimization

\[ q_0 \rightarrow \text{G2 with 0} \]
\[ q_0 \rightarrow \text{G2 with 1} \]
\[ q_1 \rightarrow \text{G2 with 0} \]
\[ q_1 \rightarrow \text{G3 with 1} \]
\[ q_2 \rightarrow \text{G2 with 0} \]
\[ q_2 \rightarrow \text{G3 with 1} \]
\[ q_3 \rightarrow \text{G3 with 0} \]
\[ q_3 \rightarrow \text{G3 with 1} \]

Groups don’t change! Thus we are done.
2) State Minimization

\[[s_0]\]: \{q_0\}  \ [s_1]\]: \{q_1, q_2\}  \ [s_2]\]: \{q_3\}

Minimized machine:
3) Non-regular Languages

Prove that \( \{0^{2n}1^n : n \geq 0\} \) is non-regular.

**Proof:** We will show that this language is non-regular by showing that there cannot exist a DFA that accepts it.

First, assume that such a DFA does exist that accepts this language, and that it has \( A \) states. (\( A \) must be a finite number because DFAs are finite— it’s even part of their name!)

Next, consider the set of strings \( \{\lambda, 00, 0000, \ldots\} \), i.e. \( \{0^{2k} : k \geq 0\} \).

This set is infinite, so we must have some strings \( 0^{2k}, 0^{2j} \) where \( k \neq j \) going to the same state. (Since our DFA has \( A \) states and \( |S| \) is infinite.)
3) Non-regular Languages

Prove that \( \{0^{2n}1^n : n \geq 0\} \) is non-regular.

Since \( 0^{2k}1^k \) is in our set, it needs to be accepted by our machine. So we must have a path of \( 1^k \) that leads to an accept state from our \( 0^{2k} \) state.

However, since \( 0^{2j} \) and \( 0^{2k} \) go to the same state, there is no way for the machine to know which path was taken, so this machine will also have to accept \( 0^{2j}1^k \). Since \( j \neq k \), this string is not in our set.

This is a contradiction, thus our assumption is false, meaning there does not exist a FSM that accepts this language. Thus our language is non-regular. \( \blacksquare \)