Problem 1:
Compute the GCD of 91 and 434 using the Euclidean Algorithm. Show the intermediate values that are computed.

Problem 2:
Use the Euclidean algorithm to solve the following problems:
a) Find an inverse of 4 modulo 21.
b) Find an inverse of 5 modulo 18.
c) Solve $13x \equiv 7 \pmod{56}$ for $x$.

Problem 3:
Prove that for every integer $n$, there are $n$ consecutive composite integers. [Hint: Consider the $n$ consecutive integers starting with $(n + 1)! + 2$.]

Problem 4:
Prove that for every positive integer $n$,
\[
\sum_{i=1}^{n} i2^i = (n - 1)2^{n+1} + 2.
\]

Problem 5:
Prove that 3 divides $n^3 + 2n$ when $n$ is a positive integer.

Problem 6:
Let $x$ be any fixed real number with $x > -1$. Prove that $(1 + x)^n \geq 1 + nx$ for every integer $n \geq 0$.

Problem 7:
Let $f_n$ be the $n$-th Fibonacci number where $f_0 = 0$, $f_1 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$. Prove that
\[
f_1^2 + f_2^2 + \cdots + f_n^2 = f_nf_{n+1}
\]
for every positive integer $n$.

Extra Credit 8:
Two integers $a$ and $b$ are relatively prime if and only if $\gcd(a, b) = 1$. Consider any $n + 1$ numbers between 1 and $2n$ (inclusive). Show that some pair of them are relatively prime.