Problems

1. Give state diagrams for (deterministic) finite state machines that recognize each of the following sets of strings. Indicate the start and final states in your diagrams and clearly label each state. In addition to the diagram, document each design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.
   a) The set of all binary strings that end with 0 and have even length, or start with 1 and have odd length.
   b) The set of all binary strings that have a 1 in every even-numbered position counting from the start of the string with the start of the string counting as position 1.

2. Give state diagrams for (deterministic) finite state machines that recognize each of the following sets of strings. Indicate the start and final states in your diagrams and clearly label each state. In addition to the diagram, document each design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.
   a) (i) The set of all binary strings that contain at least two 1's.
      (ii) The set of all binary strings that contain at most two 0's. Use different state labels from the ones you used for part (i).
      (iii) Combine the machines from parts (i) and (ii) to produce a machine that recognizes the set of all binary strings that contain at least two 1's and at most two 0's.
   b) The set of all binary strings that don't contain 001.

3. Design a finite state machine with outputs for a Candy Machine that dispenses a Gumball for 10 cents or M&M's for 15 cents. The machine takes nickels and dimes. It returns change if too much money is inserted or if the cost of the item selected is less than the amount of money deposited. A state is allowed to have multiple outputs. (Don’t worry about how the machine gets enough coins to make change.)

4. Apply the state minimization algorithm from the lectures to the FSM below. Write out the groups of states that you begin with as a sequence of sets of states. At each step, say which symbol and which group of states you are considering and how this splits the
groups of states. Show how all the states are grouped after each step. When you have finished, draw the diagram for the resulting minimized FSM.

5. Apply the construction given in lecture to convert the NFA below to a DFA that recognizes exactly the same language.

6. Draw NFAs that recognize the languages described by each of the following regular expressions. Use the construction given in lecture or in the book or produce something simpler if you can.
   a. $0110^*(110^* \cup 001^*)^*$
   b. $(00^*1)^* \cup ((11^*0)^*(0011^*)^*)^*$

Extra credit: Give a state diagram for a (deterministic) finite state machine that recognizes the set of all binary strings that represent integers that are multiples of 5 when read from left to right. Indicate the start and final states in your diagram and clearly label each state. In addition to the diagram document your design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.)