

PROOF-WRITING TIPS

- (1) Start by determining what you are given (your *hypothesis*) and what you are trying to prove (your *conclusion*). The hypothesis often appears in the statement after the words **let**, **suppose**, **assume**, or **if**. You can assume this information is true and use it freely in your proof. Conclusions often appear after the words **show** or **then**. This is the statement that you need to reach by manipulating your hypotheses and applying definitions.
- (2) In mathematical proofs, definitions are key. If you find yourself struggling with where to begin after determining your hypotheses and conclusion, look at applicable definitions and see if you can apply them to your hypotheses in order to get anything useful. For example, consider the definition of modular equivalence:

Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. Then $a \equiv b \pmod{n} \leftrightarrow n|(a - b)$.

This definition will prove helpful in many proofs involving modular equivalence.

- (3) If you are unsure of where to begin or are stuck at a certain point, trying out examples can be very useful. Be careful, though, as examples don't serve as proofs unless you are proving why a theorem is false. (These are called **counterexamples**.)
- (4) You want to write proofs in a natural, step-by-step order, like a manual. The proof should begin with stating the assumptions you are using and work logically from that point. It is very frustrating for the proof reader (i.e. the grader!) if proof steps are skipped, so even if it seems obvious, it is often best to include every step. Like a good lawyer, you want to be able to convince your reader that your proof is true beyond a reasonable doubt.
- (5) If you introduce a new letter or variable make sure you tell the reader what it is. There should be no undefined variables, just like in programming! Let the reader know if the variable is an integer, a real number, a rational number, or if it belongs to a particular set. This becomes important when applying definitions. Another common error is to introduce *too many* variables. Keep your list of characters as small as possible to help your reader remember.
- (6) Make sure you wrap up your proof by restating the conclusion. For example, "Therefore, we have shown $A \subseteq B$."
- (7) Most importantly, practice and be patient. Proofs are difficult and it takes time to become comfortable writing them. Each person will have their own proof-writing style. Aim to be clear and concise and do not worry if your proofs look different from others that you see. Always re-read over your proofs to make sure that the proof makes sense mathematically and grammatically. And don't be afraid to be a little creative!