1. Example of a subtle error in a proof by induction:

“All horses are the same color.”

You can find a pseudo-proof and an explanation in the wikipedia web page: [http://en.wikipedia.org/wiki/All_horses_are_the_same_color](http://en.wikipedia.org/wiki/All_horses_are_the_same_color)

2. "Define the Fibonacci numbers as follows: $f(0) = 0, f(1) = 1,$ and $f(n) = f(n - 2) + f(n - 1)$ for all integers $n > 1$. Prove by induction that, for all nonnegative integers $n$, the number of iterations used by Euclid’s algorithm to compute $\gcd(f(n + 1), f(n))$ is $n$.”

Proof: The basis is $n = 0$, and indeed $\gcd(1, 0)$ uses no iterations. For the induction step, the first iteration changes the arguments from $(f(n + 1), f(n))$ to $(f(n), f(n - 1))$, and the induction hypothesis says it takes $n - 1$ more iterations to finish the computation.

The only hitch is that the theorem is false for almost all values of $n$. For your entertainment, find the flaw in the proof. (It’s not hard to find once you know it’s false, but I find the proof absolutely convincing if you don’t suspect it’s false.)

3. Definition of a full binary tree:

(a) BASIS: There a binary tree with a single vertex (That vertex is also the root of the tree).

(b) RECURRENCe: Two disjoint full binary trees $T_1$ and $T_2$ can form a full binary tree. Create a new vertex as the root. Use two edges to join that root with the roots of $T_1$ and $T_2$.

Prove that every full binary tree with $k$ leaves has $k - 1$ internal vertices.

4. Prove the following:

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2} \leq 2, \ n \geq 1$$

Hint1: Try replacing the right hand side of the inequality with something that will make the statement stronger.

Hint2: Ask the TA.

5. Let $L$ denote a language using alphabet $\{0, 1\}$:
(a) BASIS: $\epsilon \in L$ (The empty string is in $L$).

(b) RECURRENCE: If $v, u \in L$ then both $0v1u$ and $1v0u$ are also in $L$.

Prove that $L$ is characterized as the collection of binary strings with an equal number of 0's and 1's. The definition of language is to be given in class.