1. Let’s learn about Cartesian products and powersets.

2. Logical equivalence with quantifiers

Determine whether the following are logically equivalent:

1. \( \forall x (P(x) \rightarrow Q(x)) \) and \( \forall x P(x) \rightarrow \forall x Q(x) \)
2. \( \exists x (P(x) \lor Q(x)) \) and \( \exists x P(x) \lor \exists x Q(x) \)

3. Use inference rules with quantified premises and conclusions

1. Premises: \( \forall x (P(x) \rightarrow (Q(x) \land S(x))) \), \( \forall x (P(x) \land R(x)) \)
   Conclusion: \( \forall x (R(x) \land S(x)) \)
2. Premises: \( \forall x (P(x) \lor Q(x)) \), \( \forall x (\neg Q(x) \lor S(x)) \), \( \forall x (R(x) \rightarrow \neg S(x)) \), \( \exists x \neg P(x) \)
   Conclusion: \( \exists x \neg R(x) \)

4. Extra: Prove that the square of a natural number \( n \) is always larger than the sum of all the numbers between 1 and \( n \) (1, \( n \) included).