2 Logical equivalence with quantifiers

Solutions in book: 7th ed- Section 1.4 Problems 43, 45; 6th ed- Section 1.3 Problems 43, 45

3 Use inference rules with quantified premises and conclusions

Solutions in book: 7th ed- Section 1.6 Problems 27, 29; 6th ed- Section 1.5 Problems 27, 29

4 Extra: Prove that the square of a natural number n is greater than or equal to the sum of all the numbers between 1 and n (1, n included).

\[ \text{Proof. Let } n \text{ be a natural number. Then the sum of all integers between 1 and } n \text{ (inclusive) can be written:} \]
\[ 1 + 2 + 3 + \ldots + n = \sum_{k=1}^{n} k. \]

Similarly, we can write \( n^2 = \sum_{k=1}^{n} k \) (i.e. \( n \) summed with itself \( n \) times).

Since \( n \geq k \) for each \( 1 \leq k \leq n \), then it follows that:
\[ \sum_{k=1}^{n} k \leq \sum_{k=1}^{n} n = n^2. \]

*Note: There are other ways to prove this. In particular, you could use theorem that states that \( \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \) and compare this with \( n^2 \).