What's on today's menu?

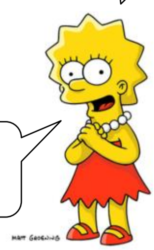
- Wrap up of Proof Techniques
- Review of Chapter 1
- Introduction to Sets

Existence Proofs

- Goal: Prove $\exists x\ P(x)$
- Two ways:

1[st] way: Constructive proof

2[nd] way: Destructive proof



2[nd] way: Non-constructive proof

Constructive Existence Proof

- Goal: Prove $\exists x\, P(x)$

Constructive proof method: Construct an a such that $P(a)$ true

Example: Prove that there exist nonzero integers x, y, z such that $x^2 + y^2 = z^2$.

Proof: Let $x = 3$, $y = 4$, $z = 5$. (Actually, infinitely many solutions)

Homework: Prove this for $x^n + y^n = z^n$ for all integers n > 2.

Scratch that. This is Fermat's last theorem: Took 358 years to prove! See > 100-pages proof by Wiles (1995).

Non-Constructive Existence Proof

- Goal: Prove $\exists x\, P(x)$

Non-constructive proof method: Prove indirectly, e.g., via a contradiction.

Example: A real no. r is rational iff $\exists$ integers p,q s.t. r = p/q. A real no. is irrational iff it is not rational. Prove that $\exists$ irrational x,y s.t. x^y is rational.

Pf. We know $\sqrt{2}$ is irrational (see text). Consider $\sqrt{2}^{\sqrt{2}}$.

Two possibilities: (a) $\sqrt{2}^{\sqrt{2}}$ is rational. Then, choose $x = y = \sqrt{2}$.

(b) $\sqrt{2}^{\sqrt{2}}$ is irrational. Choose $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$. Then, $x^y = 2$ is rational. Either way, we have shown $\exists$x,y s.t. x^y is rational.

(Doesn't say which is true!)

Review of Chapter 1

- Propositional Logic
  - Propositions, logical operators $\neg$, $\wedge$, $\vee$, $\oplus$, $\rightarrow$, $\leftrightarrow$, truth tables for operators, precedence of logical operators
  - Compound propositions, truth tables for compound propositions
  - Converse, contrapositive, and inverse of $p \rightarrow q$
  - Converting from/to English and propositional logic
- Propositional Equivalences
  - Tautology versus contradiction
  - Logical equivalence $p \equiv q$
  - Tables of logical equivalences (tables 6, 7, 8 in text)
  - De Morgan's laws
  - Showing two compound propositions are logically equivalent via (a) truth table method and (b) via equivalences in tables 6, 7, 8.

Predicate Logic

- Predicates and Quantifiers
  - Predicates, variables, and domain of each variable
  - Universal and existential quantifiers $\forall$ and $\exists$ (uniqueness $\exists!$)
  - Truth value of a quantifier statement
  - Restricting domain of a quantifier, precedence over other operators, and binding variable to a quantifier
  - Logical equivalence of two quantified statements
  - Negation and De Morgan's laws for quantifiers
  - Translating to/from English

- Nested Quantifiers
  - Quantifiers as loops
  - Order of quantifiers matters!
  - Translating to/from English, negating nested quantifiers

Rules of Inference

- Argument, Premises, Conclusion, Argument form
  - Valid argument and valid argument form (show it is a tautology).
- Rule of inference = valid argument form. Table 1 (p. 66).
  - Modus ponens: $[p \wedge (p \to q)] \to q$
  - Modus tollens: $[(p \to q) \wedge \neg q] \to \neg p$
  - Hypothetical Syllogism: $[(p \to q) \wedge (q \to r)] \to (p \to r)$
  - Disjunctive Syllogism: : $[(p \vee q) \wedge \neg p] \to q$
  - Addition, Simplification, Conjunction
  - Resolution: $[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$
- Using rules of inference to prove statements from premises
- Rules of inference for quantified statements: instantiation and generalization

Proofs and Proof Methods

- Direct proof of $p \to q$: Assume p is true; show q is true.
  - Example in class: If n is an even integer, then n^2 is even.
- Proof of $p \to q$ by contraposition: Assume $\neg q$ and show $\neg p$.
  - Example in class: If n^2 is even for integer n, then n is even.
- Vacuous and Trivial Proofs of $p \to q$
- Proof by contradiction of a statement p: Assume p is not true and show this leads to a contradiction $(r \wedge \neg r)$.
  - Example in class: Pigeonhole principle
- Proofs of equivalence for $p \leftrightarrow q$: Show $p \to q$ and $q \to p$
- Proof by cases and Existence proofs

Enuff review,
let's move on to sets!!