Final exam review

The final exam will be Monday, June 6, 2:30pm-4:20pm. You may bring two double-sided sheets of notes on 8 1/2” x 11” paper. Roughly 1/3 will be on material before the midterm and 2/3 on material since the midterm.

Below are a list of topics that you should know, along with suggested odd-numbered exercises if you want extra practice. The notation \([x.y : z_1, z_2, \ldots]\) means exercises \(z_1, z_2, \ldots\) in section \(x.y\). You don’t have to do any of these problems if you feel confident about your knowledge of the material.

**Chapter 1:**
- Converting between propositions/predicates and English/mathematical statements. [1.1:9,17]
- Logical equivalences [1.2:27,33,57]
- Inference and logical equivalences [1.5:15,19,27]
- Proof strategies (direct, contrapositive, contradiction, cases, WLOG) [1.6:3,5,13,17,35,39; 1.7:3,5,13,15]

**Chapter 2:**
- sets
  - set builder notation [2.1:5]
  - subsets, empty set [2.1:7,9], [2.1:15, but prove only using the definition of \(\subseteq\) and not the set identities in Section 2.2]
  - power set [Define a function \(f\) whose domain is finite sets by \(f(S) = P(S)\). Construct the inverse \(f^{-1}\), i.e. a function such that \(f^{-1}(f(S)) = S\) for any finite set \(S\).]
  - Cartesian product [2.1:27,29]
  - intersection, union, complement, set difference: [2.2:19,25,29]
  - disjoint sets
  - cardinality and infinite sets. [Prove or disprove: If \(A\) and \(B\) are finite sets, then \(|A \cup B| \leq |A| + |B|.|]
  - using set notation to understand other parts of the course: quantifiers, number theory, induction, etc.
- functions
  - terminology (domain/codomain/range/image/preimage). [2.3:5]
  - properties (onto/surjective, 1-1/injective, bijective). [2.3:19,21,31]
  - methods of proving a function has these properties (either directly or using the fact that function is increasing or decreasing). [2.3:23,35]

**Chapter 3:**
- divisibility [3.4:3,5,7]
• division algorithm [3.4:9ab]
• modular arithmetic [3.4:11,21,23]
• primes [3.5:9,35]
• relatively prime [3.5:11]
• FTA, GCD, LCM [3.5:27]
• multiplicative inverses, solving linear congruences [3.7:13,15]
• Fermat’s little theorem [3.7:17]

Chapter 4:
• induction [4.1:15 (proving an equality), 21 (proving an inequality), 39 (proofs about sets), 49]
• strong and structural induction [4.2:3,13,29]
• variants of induction [4.2:25]
• recursive definitions [4.3:27,37]
• recursive algorithms [4.4:23]

Chapter 8:
• properties of relations (symmetry, antisymmetry, reflexivity, transitivity) [8.1:5,7]
• equivalence relations and partitions [8.5:3,15,45]
• partial orders [8.6:1,3,5,37]

Chapters 9 and 10:
• graph types (directed/undirected, self-loops/multiple-edges allowed or not, weighted or unweighted) [9.1:3,5,7,9 (but don’t worry about the table),11]
• degree [9.2:15]
• paths, circuits and connectedness in undirected graphs [9.4:1, 27]
• tree definition [10.1:1]
• quantitative features of trees [10.1:15,23,37]

Chapter 11:
• Boolean algebra [11.1:3, 5a, 33]
• sum-of-products expansion [11.2:5]
• functional completeness [11.2:19]
• circuits [11.3:1,3, 11, 15]

Chapter 12:
• finite state machines with output [12.2: 1a, 3a, 9]
• finite state machines without output [12.3: 17, 27]
• nondeterministic finite automata and converting them to DFAs [12.3:51]
• regular sets and expressions [12.4:3,5,11]
• recognizing regular sets with NFAs [12.4:13]
• Turing machines [12.5:3,7,15]

Topics that will not be on the exam:
• truth tables
• rings and fields
• Chinese Remainder Theorem
• RSA
• triominoes
• induction on general well-ordered sets
• graph notions not listed above (e.g. strongly connected, Eulerian cycle, etc.)
• infinite graphs
• deciding whether to use weak/strong/structural induction (the question will specify which type of induction to use)