CSE 311: Foundations of Computing I
Assignment #7
due: Fri, May 27, 1:30pm

1. *(15 points)* Section 8.5, exercise 16
2. *(10 points)* Define a relation R on $\mathbb{R} \times \mathbb{R}$ by $((a, b), (c, d)) \in R$ if $(a < c) \wedge (b < d)$. Is this a partial order, a total order, or neither? Justify your answer.
3. *(10 points)* Following Definition 1, consider the poset $(\mathbb{R}^+, \leq)$, where $\mathbb{R}^+ = \{x \in \mathbb{R} | x > 0\}$. Is $(\mathbb{R}^+, \leq)$ well ordered? Justify your answer.
4. *(10 points)* Let $G = (V, E)$ be an undirected graph with no self-loops or multiple edges. For $v \in V$, define $\deg(v)$ to be the degree of v. Find a rule relating $|E|$ to $\sum_{v \in V} \deg(v)$, and prove your answer.
5. *(15 points)* Prove that any tree with at least two vertices must contain at least one vertex of degree one.
6. *(6 points)* Section 10.1. Exercise 2
7. *(14 points)* Section 10.1. Exercise 14
8. *(8 points)* Section 11.1. Exercise 6d
9. *(8 points)* Section 11.2. Exercise 4, all parts.
10. *(14 points)* Section 11.3. Exercise 10. Gates can be drawn either using the symbols in the book, or by writing AND, OR and NOT.

Additional optional problems: These problems are suggested for those looking for practice problems for the final exam. If you are confident about your understanding of the subject, then feel free to ignore them.

1. Section 8.1. Exercises 5, 7
2. Section 8.5. Exercises 3, 15, 45
3. Section 8.6. Exercises 1, 3, 5, 37
4. Chapter 8. Review question 13
5. Section 9.1. Exercises 11, 13
6. Section 10.1. Exercises 15, 23 (please don't get any ideas from this), 31
7. Section 11.1. Exercises 3, 5a
8. Section 11.2. Exercise 5
9. Section 11.3. Exercises 11, 15
10. *(15 points)* Let $G = (V, E)$ be a directed graph, possibly with self-loops. Let $V = \{1, \ldots, n\}$. The adjacency matrix A is defined by setting $A_{i,j} = 1$ if $(i, j) \in E$ and $A_{i,j} = 0$ if $(i, j) \notin E$. For $t \in \mathbb{N}$, we write A^t for $\overbrace{A \cdot A \cdots A}^{t \text{ times}}$, where $\cdot$ means matrix multiplication. Prove that $(A^t)_{i,j}$ counts the number of paths of length t that start at i and end at j. *Hint: use induction.* See section 9.4 (page 628) for solution.