CSE 311: Foundations of Computing I Assignment #6 due: Fri, May 20, 1:30pm

- 1. (15 points) Section 4.3, exercise 26, all parts
- 2. (10 points) Section 4.4, exercise 10
- 3. (10 points) Section 4.4, exercise 22
- 4. (5 points) Chapter 4, Supplementary exercise 12(a) (on page 330)
- 5. (16 points) Section 8.1, exercise 6, all parts. Your answer should be in the form of a table with Y/N in each entry. You don't need to justify your answers.
- 6. (10 points) Section 8.1, exercise 8, both parts
- 7. (10 points) Section 8.1, exercise 14
- 8. (24 points) Let p be a prime, and let  $S := [p-1] = \{1, \ldots, p-1\}$ . For any  $a \in S$ , define the relation R on S by  $\{(x, y) : x, y \in S \land \exists j \in \mathbb{N}(xa^j \equiv y \pmod{p})\}$ . In parts (b) and (c), define r to be the smallest positive integer satisfying  $a^r \equiv 1 \pmod{p}$ .
  - (a) Prove that R is an equivalence relation.
  - (b) Prove that for  $x \in S$ , the equivalence class [x] is equal to  $\{x, ax \mod p, a^2x \mod p, \dots, a^{r-1}x \mod p\} = \{xa^j \mod p : 0 \le j < r\}.$
  - (c) Prove that r|p-1. What can you conclude about  $a^{p-1} \mod p$ ?