CSE 311: Foundations of Computing I
Assignment #4
due: Fri, Apr 29, 1:30pm

1. *(15 points)* Section 3.4, exercise 14
2. *(15 points)* Section 3.5, exercise 18
3. *(15 points)* Define $\phi(n)$ as in the previous problem. Let p_1, p_2, p_3 be distinct primes.
   (a) Calculate $\phi(15)$.
   (b) State and prove a formula for $\phi(p_1 p_2)$. *Hint: use inclusion-exclusion.*
   (c) State and prove a formula for $\phi(p_1 p_2 p_3)$.
4. Euclid's algorithm:
   (a) *(5 points)* Section 3.6, exercise 24c
   (b) *(5 points)* Find integers m, n such that $123m + 277n = \gcd(123, 277)$.
5. *(10 points)* In Western music, the octave is divided into 12 semitones. Normally they are named A, A$^\sharp$, B, C, C$^\sharp$, D, D$^\sharp$, E, F, F$^\sharp$, G, G$^\sharp$, but for this problem, call them $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$. A "perfect fifth" is an interval of 7 semitones. When we reach the top of the scale ($G^\sharp$ or 11), we repeat from the bottom (A or 0). This means that a perfect fifth starting on note x will end on note $x + 7 \bmod 12$.
   (a) If we start on note x and ascend by perfect fifths, we will encounter x, $x+7 \bmod 12$, $x+14 \bmod 12$, etc. How many perfect fifths do we need to ascend by until we come back to note x?
   (b) The dissonant "tritone" is an interval of 6 semitones. If we start with note x, then how many tritones do we need to ascend by until we come back to x?
6. *(10 points)* Solve the congruence $14x \equiv 7 \pmod{17}$.
7. *(10 points)* Section 3.7, exercise 14 (both parts)
8. *(15 points)* Suppose that x, y, n are all positive integers with $n > 1$. Suppose that $x \not\equiv 0 \pmod{n}$ and $y \not\equiv 0 \pmod{n}$ but $xy \equiv 0 \pmod{n}$. Prove that n must be composite.