1. (15 points) Section 3.4, exercise 14

2. (15 points) Section 3.5, exercise 18

3. (15 points) Define $\phi(n)$ as in the previous problem. Let $p_1, p_2, p_3$ be distinct primes.
   (a) Calculate $\phi(15)$.
   (b) State and prove a formula for $\phi(p_1 p_2)$. \emph{Hint: use inclusion-exclusion.}
   (c) State and prove a formula for $\phi(p_1 p_2 p_3)$.

4. Euclid’s algorithm:
   (a) (5 points) Section 3.6, exercise 24c
   (b) (5 points) Find integers $m, n$ such that $123m + 277n = \gcd(123, 277)$.

5. (10 points) In Western music, the octave is divided into 12 semitones. Normally they are named A, A♯, B, C, C♯, D, D♯, E, F, F♯, G, G♯, but for this problem, call them \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}. A “perfect fifth” is an interval of 7 semitones. When we reach the top of the scale (G♯ or 11), we repeat from the bottom (A or 0). This means that a perfect fifth starting on note $x$ will end on note $x + 7 \pmod{12}$.
   (a) If we start on note $x$ and ascend by perfect fifths, we will encounter $x, x + 7 \pmod{12}, x + 14 \pmod{12}$, etc. How many perfect fifths do we need to ascend by until we come back to note $x$?
   (b) The dissonant “tritone” is an interval of 6 semitones. If we start with note $x$, then how many tritones do we need to ascend by until we come back to $x$?

6. (10 points) Solve the congruence $14x \equiv 7 \pmod{17}$.

7. (10 points) Section 3.7, exercise 14 (both parts)

8. (15 points) Suppose that $x, y, n$ are all positive integers with $n > 1$. Suppose that $x \not\equiv 0 \pmod{n}$ and $y \not\equiv 0 \pmod{n}$ but $xy \equiv 0 \pmod{n}$. Prove that $n$ must be composite.