Announcements

• Reading assignments
• Upcoming topics
  – Relations
  – Finite State Machines

Definition of Relations

Let A and B be sets,
A binary relation from A to B is a subset of \( A \times B \)

Let A be a set,
A binary relation on A is a subset of \( A \times A \)

Relation Examples

\( R_1 = \{ (a, 1), (a, 2), (b, 1), (b, 3), (c, 3) \} \)
\( R_2 = \{ (x, y) \mid x \equiv y \text{ (mod 5)} \} \)
\( R_3 = \{ (c_1, c_2) \mid c_1 \text{ is a prerequisite of } c_2 \} \)
\( R_4 = \{ (s, c) \mid \text{ student } s \text{ had taken course } c \} \)

Properties of Relations

Let R be a relation on A

R is reflexive iff \((a, a) \in R\) for every \(a \in A\)

R is symmetric iff \((a, b) \in R\) implies \((b, a) \in R\)

R is antisymmetric iff \((a, b) \in R\) and \(a \neq b\) implies \((b, a) \notin R\)

R is transitive iff \((a, b) \in R\) and \((b, c) \in R\) implies \((a, c) \in R\)

Combining Relations

Let R be a relation from A to B
Let S be a relation from B to C
The composite of R and S, \( S \circ R \) is the relation from A to C defined

\( S \circ R = \{ (a, c) \mid \exists b \text{ such that } (a, b) \in R \text{ and } (b, c) \in S \} \)
Examples

(a,b) ∈ Parent: b is a parent of a
(a,b) ∈ Sister: b is a sister of a

What is Parent ∘ Sister?

What is Sister ∘ Parent?

$S ∘ R = \{(a, c) \mid \exists b \text{ such that } (a, b) \in R \text{ and } (b, c) \in S\}$

Examples

Using the relations: Parent, Child, Brother, Sister, Sibling, Father, Mother express

Uncle: b is an uncle of a
Cousin: b is a cousin of a

Powers of a Relation

$R^2 = R ∘ R = \{(a, c) \mid \exists b \text{ such that } (a, b) \in R \text{ and } (b, c) \in R\}$

$R^0 = \{(a, a) \mid a \in A\}$

$R^1 = R$

$R^{n+1} = R^n ∘ R$

How is Anderson related to Bernoulli?

http://genealogy.math.ndsu.nodak.edu/
Let $A_1, A_2, \ldots, A_n$ be sets. An $n$-ary relation on these sets is a subset of $A_1 \times A_2 \times \ldots \times A_n$.
Database Operations

Projection

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