1. Possibly helpful tools on the textbook website, www.mhhe.com/rosen
   - Interactive Demonstration Applets
     - Truth Tables
     - Equivalences
   - Self Assessments
     - Conditional Statements
     - Quantified Statements
   - Guide to Writing Proofs
   - Common Mistakes

2. Prove that $(p \rightarrow r) \land (q \rightarrow r) \equiv (p \lor q) \rightarrow r$ by rewriting with equivalences.

3. Prove that $(p \land q) \rightarrow (p \rightarrow q)$ is a tautology by rewriting with equivalences.

4. Find the values, if any, of the Boolean variable $x$ that satisfies these equations:
   
   (a) $x \cdot 1 = 0$
   (b) $x + x = 0$
   (c) $x \cdot 1 = x$
   (d) $x \cdot \bar{x} = 1$

5. Use truth tables to express the values of these Boolean functions:
   
   (a) $F(x, y, z) = \overline{xy} + \overline{xz}$
   (b) $F(x, y, z) = \overline{y}(xz + \overline{xz})$

6. For a Boolean function on each of the following number of inputs:
   
   - How many rows are in the truth table?
   - How many different Boolean functions are possible?
     - 3 inputs ("a Boolean function of degree 3")
     - 4 inputs
7. Half adder

(a) Write the truth table for a half adder (takes two bits, \(x\) and \(y\), and outputs two bits - \(s\) (sum) and \(c\) (carry):

(b) Use the truth table to write the boolean expressions for outputs \(s\) and \(c\). (Don’t minimize.)

(c) How many gates will you need in a circuit that implements these expressions?

(d) Draw the circuit.

(e) Minimize the expression for output \(s\). Now how many gates do you need?

(f) Draw the simplified circuit.

8. Repeat the steps from the above problem (using \(t\) as the single output value) for the Boolean function given by the following truth table:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(z)</th>
<th>(t)</th>
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<tbody>
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