In problems 1 and 2, \( f_n \) is the \( n \)th Fibonacci number where \( f_0 = 0 \), \( f_1 = 1 \) and \( f_k = f_{k-1} + f_{k-2} \) for \( k \geq 2 \).

**Problem 1:**
Prove that \( f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1} \) when \( n \) is a positive integer.

**Problem 2:**
Let
\[
A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}
\]
prove that
\[
A^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}
\]
when \( n \) is a positive integer.

**Problem 3:**
Give a recursive definition of
a) The set of integers that are congruent to 1 or 3 modulo 7.

b) The set of polynomials in \( x \) with integer coefficients.

**Problem 4:**
Give a recursive definition of the set of bit strings that have the same number of zeros and ones.

**Problem 5:**
Give a recursive definition of the following set of ordered pairs of positive integers:
\[
S = \{(a, b) \mid a \in \mathbb{Z}^+, b \in \mathbb{Z}^+, \text{and } a + b \text{ is odd}\} \]