Problem 1:
Suppose that \( \mathcal{P}(A) = \mathcal{P}(B) \). Show that \( A = B \).

Problem 2:
Suppose that \( C \neq \emptyset \) and \( A \times C = B \times C \). Show that \( A = B \). Why do we need the assumption that \( C \neq \emptyset \)?

Problem 3:
Which, if any, of the following assumptions implies that \( A = B \) for all sets \( A, B, \) and \( C \)? Justify your answers.
(a) \( A \cup C = B \cup C \).
(b) \( A \cap C = B \cap C \).
(c) Both \( A \cup C = B \cup C \) and \( A \cap C = B \cap C \).

Problem 4:
Prove that if \( n \) is an integer then \( n^2 \mod 5 \) is either 0, 1, or 4.

Problem 5:
Let \( a, b \) be integers and \( c, n \) be positive integers.
Prove that if \( a \equiv b \pmod{n} \) then \( ac \equiv bc \pmod{cn} \).

Problem 6:
Let \( E_7^3 = \{ x \mid x \equiv 3 \pmod{7} \} \) and \( E_{21}^{10} = \{ x \mid x \equiv 10 \pmod{21} \} \).
Prove \( E_{21}^{10} \subseteq E_7^3 \).

Problem 7:
For each \( a \in \{1, \ldots, 10\} \) determine the smallest \( k \geq 1 \) such that \( a^k \mod 11 = 1 \).

Problem 8:
How many multiplications are needed to compute \( 37^{1000} \mod 10000 \) using the fast modular exponentiation algorithm? (You do not need to compute \( 37^{1000} \mod 10000 \), just determine the number of multiplications and justify your answer.)
Extra Credit 9:

(a) Use a formula for the unsigned decimal expansion of integers to justify the “casting out nines” rule that a positive number is divisible by 9 if and only if repeatedly adding its digits gives 9. (For example, the digit sum for 729 is 18 which has digit sum 9.)

(b) Do the same for the “casting out threes” rule which says that a number is divisible by 3 if and only if the final digit sum is one of 3, 6, or 9.