Problem 1:
Show that \((p \rightarrow q) \land (q \rightarrow r) \rightarrow (p \rightarrow r)\) is a tautology by applying a series of equivalences to derive T.

Problem 2:
Show that \((p \land q) \rightarrow r\) and \((p \rightarrow r) \land (q \rightarrow r)\) are not equivalent.

Problem 3:
Find a compound proposition involving the propositional variables \(p, q,\) and \(r\), that is true when \(p\) and \(q\) are true and \(r\) is false, but is false otherwise.

Problem 4:
Use truth tables to represent the values of each of these boolean functions:
   a) \(F(x, y, z) = \overline{xyz} + (xyz)\)
   b) \(F(x, y, z) = x(yz + \overline{y} \overline{z})\)

Problem 5:
Find the sum-of-products expansion of these Boolean functions:
   a) \(F(x, y, z) = (x + z)y\)
   b) \(F(x, y, z) = x\)
   c) \(F(x, y, z) = x\overline{y}\)

Problem 6:
Construct a circuit that computes the product of the two two-bit integers \((x_1x_0)_2\) and \((y_1y_0)_2\). The circuit should have four output bits for the bits in the product.

Problem 7:
Translate these statements into English, where \(R(x)\) is "\(x\) is a rabbit" and \(H(x)\) is "\(x\) hops" and the domain consists of all animals.
   a) \(\forall x (R(x) \land H(x))\)
   b) \(\exists x (R(x) \rightarrow H(x))\)
Problem 8:
Let $C(x)$ be the statement “$x$ has a cat”, $D(x)$ be the statement “$x$ has a dog”, and $F(x)$ be the statement “$x$ has a ferret”. Express each of the following statements using quantifiers, logical connectives, and the above statements. The domain consists of all students in the class.
a) All students in the class have a cat, a dog, and a ferret.
b) Some student in the class has a cat and a ferret, but not a dog.
c) For each of the three animals, cats, dogs, and ferrets, there is a student in the class who has one of these animals as a pet.

Extra Credit 9:
Design a boolean circuit that has six inputs, and one output where the output is 1 if exactly three of the inputs are 1, and is 0 otherwise. Your circuit should use as few gates as possible. Provide a brief explanation as to how your circuit works. [I don’t know what the best possible result is, I came up with a circuit that uses 13 AND gates, 3 XOR gates, 1 OR gate, and 6 NOT Gates.]