Functions and abstraction

Ruth Anderson

UW CSE 160

Winter 2016
Functions

• In math, you use functions: sine, cosine, ...
• In math, you define functions: \( f(x) = x^2 + 2x + 1 \)

• A function packages up and names a computation
• Enables re-use of the computation (generalization)
• Don’t Repeat Yourself (DRY principle)
• Shorter, easier to understand, less error-prone

• Python lets you use and define functions
• We have already seen some Python functions:
  – len, float, int, str, range
Using ("calling") a function

len("hello")    len(""")
round(2.718)     round(3.14)
pow(2, 3)        range(1, 5)
math.sin(0)      math.sin(math.pi / 2)

• Some need no input: random.random()
• All produce output
• What happens if you forget the parentheses on a function call? random.random
  – Functions are values too
  – Types we know about: int, float, str, bool, list, function
A function is a machine

• You give it input
• It produces a result

In math: $\text{func}(x) = 2x + 1$
Creating a function

Define the machine, including the input and the result

```python
def dbl_plus(x):
    return 2*x + 1
```

Keyword that means: I am defining a function

Name of the function. Like “y = 5” for a variable

Input variable name, or “formal parameter”

Keyword that means: This is the result

Return expression (part of the `return` statement)
More function examples
Define the machine, including the input and the result

```python
def square(x):
    return x * x

def fahr_to_cent(fahr):
    return (fahr - 32) / 9.0 * 5

def cent_to_fahr(cent):
    result = cent / 5.0 * 9 + 32
    return result

def abs(x):
    if x < 0:
        return -x
    else:
        return x

def print_hello():
    print "Hello, world"

def print_fahr_to_cent(fahr):
    result = fahr_to_cent(fahr)
    print result
```

What is the result of:
```
x = 42
square(3) + square(4)
print x
boiling = fahr_to_cent(212)
cold = cent_to_fahr(-40)
print result
print abs(-22)
print print_fahr_to_cent(32)
```
Digression: Two types of output

• An expression evaluates to a value
  – Which can be used by the containing expression or statement
• A `print` statement writes text to the screen

• The Python `interpreter` (command shell) reads statements and expressions, then executes them
• If the `interpreter` executes an expression, it prints its value
• In a `program`, evaluating an expression does not print it
• In a `program`, printing an expression does not permit it to be used elsewhere
How Python executes a function call

1. Evaluate the argument (at the call site)
2. Assign the actual argument’s value to the formal parameter name
   – A new variable, not reuse of any existing variable of the same name
3. Evaluate the statements in the body one by one
4. At a return statement:
   – Remember the value of the expression
   – Formal parameter variable disappears – exists only during the call!
   – The call expression evaluates to the return value

```python
def square(x):
    return x * x
```

Square (3 + 4)

Current expression:
1 + square(3 + 4)
1 + square(7)
evaluate this expression
1 + 49
50
Example of function invocation

def square(x):
    return x * x

square(3) + square(4)
return x * x
return 3 * x
return 3 * 3
return 9
9 + square(4)
   return x * x
   return 4 * x
   return 4 * 4
return 16
9 + 16
25

Variables:
(none)
x: 3
x: 3
x: 3

(none)
x: 4
x: 4
x: 4

(none)
(none)
Expression with nested function invocations: Only one executes at a time

```python
def fahr_to_cent(fahr):
    return (fahr - 32) / 9.0 * 5

def cent_to_fahr(cent):
    return cent / 5.0 * 9 + 32

fahr_to_cent(cent_to_fahr(20))
    return cent / 5.0 * 9 + 32
    return 20 / 5.0 * 9 + 32
    return 68

fahr_to_cent(68)
    return (fahr - 32) / 9.0 * 5
    return (68 - 32) / 9.0 * 5
    return 20

20
```

Variables:

- cent: 20
- cent: 20
- cent: 20

- fahr: 68
- fahr: 68
- fahr: 68

- (none)
Expression with nested function invocations: Only one executes at a time

```python
def square(x):
    return x * x

square(square(3))
    return x * x
    return 3 * x
    return 3 * 3
    return 9

square(9)
    return x * x
    return 9 * x
    return 9 * 9
    return 81

81
```

Variables:

<table>
<thead>
<tr>
<th>Function Call</th>
<th>Arguments</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>square(square(3))</td>
<td>(none)</td>
<td>x: 3, x: 3, x: 3</td>
</tr>
<tr>
<td>square(9)</td>
<td>(none)</td>
<td>x: 9, x: 9, x: 9</td>
</tr>
</tbody>
</table>

11
Function that invokes another function:
Both function invocations are active

def square(z):
    return z*z
def hypotenuse(x, y):
    return math.sqrt(square(x) + square(y))

hypotenuse(3, 4)
    return math.sqrt(square(x) + square(y))
    return math.sqrt(square(3) + square(y))
        return z*z
        return 3*3
        return 9
    return math.sqrt(9 + square(y))
    return math.sqrt(9 + square(4))
        return z*z
        return 4*4
        return 16
    return math.sqrt(9 + 16)
return math.sqrt(25)
return 5

Variables:
(none)
x: 3   y:4
x: 3   y:4
z: 3   x: 3   y:4
z: 3   x: 3   y:4
z: 3   x: 3   y:4
x: 3   y:4
x: 3   y:4
z: 4   x: 3   y:4
z: 4   x: 3   y:4
z: 4   x: 3   y:4
x: 3   y:4
x: 3   y:4
(none)
Shadowing of formal variable names

```python
def square(x):
    return x*x

def hypotenuse(x, y):
    return math.sqrt(square(x) + square(y))

hypotenuse(3, 4)
    return math.sqrt(square(x) + square(y))
    return x*x
    return 3*3
    return 9
return math.sqrt(9 + square(y))
return math.sqrt(9 + square(4))
    return x*x
    return 4*4
    return 16
return math.sqrt(9 + 16)
return math.sqrt(25)
return 5
```

**Variables:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>3</td>
</tr>
<tr>
<td>y</td>
<td>4</td>
</tr>
</tbody>
</table>

Formal parameter is a *new* variable.
Shadowing of formal variable names

def square(x):
    return x**x

def hypotenuse(x, y):
    return math.sqrt(square(x) + square(y))

hypotenuse(3, 4)
    return math.sqrt(square(x) + square(y))
    return math.sqrt(square(3) + square(y))
        return x**x
        return 3*3
        return 9
    return math.sqrt(9 + square(y))
    return math.sqrt(9 + square(4))
        return x**x
        return 4*4
        return 16
    return math.sqrt(9 + 16)
    return math.sqrt(25)
    return 5

Same diagram, with variable scopes or environment frames shown explicitly

Variables:

<table>
<thead>
<tr>
<th>hypotenuse()</th>
<th>x: 3</th>
<th>y: 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>square()</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>(none)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x: 3</td>
<td>x: 3</td>
<td>y: 4</td>
</tr>
<tr>
<td>x: 4</td>
<td>x: 4</td>
<td></td>
</tr>
<tr>
<td>x: 4</td>
<td>x: 4</td>
<td></td>
</tr>
<tr>
<td>(none)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In a function body, assignment creates a temporary variable (like the formal parameter)

```
stored = 0
def store_it(arg):
    stored = arg
    return stored

y = store_it(22)
print y
print stored
```

Show evaluation of the starred expressions:
```
y = store_it(22)
    stored = arg
    stored = 22
    return stored
    return 22
```
```
y = 22
print stored
print 0
```
How to look up a variable

Idea: find the nearest variable of the given name

1. Check whether the variable is defined in the local scope
2. ... check any intermediate scopes (none in CSE 160!) ...
3. Check whether the variable is defined in the global scope

If a local and a global variable have the same name, the global variable is inaccessible (“shadowed”)
This is confusing; try to avoid such shadowing

```
x = 22
stored = 100
def lookup():
    x = 42
    return stored + x
lookup()
x = 5
stored = 200
lookup()
```

```
def lookup():
    x = 42
    return stored + x
lookup()
x = 22
stored = 100
lookup()
x = 5
stored = 200
lookup()
```

What happens if we define `stored` after `lookup`?
Local variables exist only while the function is executing

def cent_to_fahr(cent):
    result = cent / 5.0 * 9 + 32
    return result

tempf = cent_to_fahr(15)
print result
Use only the local and the global scope

```python
myvar = 1

def outer():
    myvar = 1000
    return inner()

def inner():
    return myvar

print outer()
```

Aside: The Evaluation Rules have a more precise rule, which applies when you define a function inside another function (which we will not be doing in this class).
Abstraction

• Abstraction = ignore some details
• Generalization = become usable in more contexts
• Abstraction over computations:
  – functional abstraction, a.k.a. procedural abstraction
• As long as you know what the function means, you don’t care how it computes that value
  – You don’t care about the implementation (the function body)
def abs(x):
    if val < 0:
        return -1 * val
    else:
        return 1 * val

def abs(x):
    result = val
    return result

def abs(x):
    if val < 0:
        result = - val
    else:
        result = val
    return result

def abs(x):
    return math.sqrt(x*x)
Defining round
(for positive numbers)

def round(x):
    return int(x+0.5)

def round(x):
    fraction = x - int(x)
    if fraction >= .5:
        return int(x) + 1
    else:
        return int(x)
# Two types of documentation

1. Documentation for users/clients/callers
   - Document the *purpose* or *meaning* or *abstraction* that the function represents
   - Tells *what* the function does
   - Should be written for every function

2. Documentation for programmers who are reading the code
   - Document the *implementation* – specific code choices
   - Tells *how* the function does it
   - Only necessary for tricky or interesting bits of the code

```python
def square(x):
    """Returns the square of its argument."""
    # Uses "x*x" instead of "x**2"
    return x*x
```
Multi-line strings

• New way to write a string – surrounded by three quotes instead of just one
  – "hello"
  – 'hello'
  – """hello""
  – '''hello'''

• Any of these works for a documentation string

• Triple-quote version:
  – can include newlines (carriage returns), so the string can span multiple lines
  – can include quotation marks
Don’t write useless comments

• Comments should give information that is not apparent from the code
• Here is a counter-productive comment that merely clutters the code, which makes the code harder to read:

```python
# increment the value of x
x = x + 1
```

DO NOT write comments like this.
Where to write comments

- By convention, write a comment *above* the code that it describes (or, more rarely, on the same line)
  - First, a reader sees the English intuition or explanation, then the possibly-confusing code
    # The following code is adapted from
    # “Introduction to Algorithms”, by Cormen et al.,
    # section 14.22.
    while (n > i):
      ...
- A comment may appear anywhere in your program, including at the end of a line:
  x = y + x    # a comment about this line
- For a line that starts with #, indentation should be consistent with surrounding code
Each variable should represent one thing

```python
def atm_to_mbar(pressure):
    return pressure * 1013.25

def mbar_to_mmHg(pressure):
    return pressure * 0.75006

# Confusing
pressure = 1.2  # in atmospheres
pressure = atm_to_mbar(pressure)
pressure = mbar_to_mmHg(pressure)
print pressure

# Better
in_atm = 1.2
in_mbar = atm_to_mbar(in_atm)
in_mmHg = mbar_to_mmHg(in_mbar)
print in_mmHg

# Best

def atm_to_mmHg(pressure):
    in_mbar = atm_to_mbar(pressure)
in_mmHg = mbar_to_mmHg(in_mbar)
    return in_mmHg

print atm_to_mmHg(1.2)
```

Corollary: Each variable should contain values of only one type

```python
# Legal, but confusing: don’t do this!
x = 3
...
x = "hello"
...
x = [3, 1, 4, 1, 5]
...
```

If you use a descriptive variable name, you are unlikely to make these mistakes
Exercises

```python
def cent_to_fahr(cent):
    print cent / 5.0 * 9 + 32

print cent_to_fahr(20)

def c_to_f(c):
    print "c_to_f"
    return c / 5.0 * 9 + 32

def make_message(temp):
    print "make_message"
    return ("The temperature is " + str(temp))

for tempc in [-40,0,37]:
    tempf = c_to_f(tempc)
    message = make_message(tempf)
    print message

def myfunc(n):
    total = 0
    for i in range(n):
        total = total + i
    return total

print myfunc(4)

Use the Python Tutor: http://pythontutor.com/
```
What does this print?

```python
def cent_to_fahr(cent):
    print cent / 5.0 * 9 + 32

print cent_to_fahr(20)
```
What does this print?

```python
def myfunc(n):
    total = 0
    for i in range(n):
        total = total + i
    return total

print myfunc(4)
```
def c_to_f(c):
    print "c_to_f"
    return c / 5.0 * 9 + 32

def make_message(temp):
    print "make_message"
    return "The temperature is " + str(temp)

for tempc in [-40,0,37]:
    tempf = c_to_f(tempc)
    message = make_message(tempf)
    print message
Decomposing a problem

• Breaking down a program into functions is the fundamental activity of programming!

• How do you decide when to use a function?
  – One rule: DRY (Don’t Repeat Yourself)
  – Whenever you are tempted to copy and paste code, don’t!

• Now, how do you design a function?
How to design a function

1. Wishful thinking:
   Write the program as if the function already exists

2. Write a specification:
   Describe the inputs and output, including their types
   No implementation yet!

3. Write tests: Example inputs and outputs

4. Write the function body (the implementation)
   First, write your plan in English, then translate to Python

```python
def fahr_to_cent(fahr):
    """Input: a number representing degrees Farenheit
    Return value: a number representing degrees centigrade
    """
    result = (fahr - 32) / 9.0 * 5
    return result

assert fahr_to_cent(32) == 0
assert fahr_to_cent(212) == 100
assert fahr_to_cent(98.6) == 37
assert fahr_to_cent(-40) == -40

# Main program
tempf = 32
print "Temperature in Farenheit:", tempf
tempc = fahr_to_cent(tempf)
print "Temperature in Celsius:", tempc
```
Review: how to evaluate a function call

1. Evaluate the function and its arguments to values
   – If the function value is not a function, execution terminates with an error
2. Create a new stack frame
   – The parent frame is the one where the function is defined
     • In CSE 160, this is always the global frame
   – A frame has bindings from variables to values
   – Looking up a variable starts here
     • Proceeds to the next older frame if no match here
     • The oldest frame is the “global” frame
     • All the frames together are called the “environment”
   – Assignments happen here
3. Assign the actual argument values to the formal parameter variable
   – In the new stack frame
4. Evaluate the body
   – At a return statement, remember the value and exit
   – If at end of the body, return None
5. Remove the stack frame
6. The call evaluates to the returned value
Functions are values
The function can be an expression

def double(x):
    return 2*x
def doubler():
    return double

print double
def doubler():
    return double

myfns = [math.sqrt, int, double, math.cos]
print myfns[1](3.14)
print myfns[2](3.14)
print myfns[3](3.14)
print doubler()(2.718)