1. Inheritance and Polymorphism.

Consider the following classes (System.out.println has been abbreviated as S.o.pln):

```java
public class Leo extends Don {
    public void method1() {
        S.o.pln("Leo 1");
    }

    public void method3() {
        S.o.pln("Leo 3");
        method1();
    }
}

public class Mike extends Leo {
    public void method2() {
        S.o.pln("Mike 2");
        super.method2();
    }
}

public class Raph {
    public void method1() {
        S.o.pln("Raph 1");
    }
}

public class Don extends Raph {
    public void method2() {
        method1();
        S.o.pln("Don 2");
    }
}
```

The following variables are defined:

- Raph var1 = new Don();
- Leo var2 = new Mike();
- Object var3 = new Raph();
- Don var4 = new Leo();

In the table below, indicate in the right-hand column the output produced by the statement in the left-hand column. If the statement produces more than one line of output, indicate the line breaks with slashes as in "a / b / c" to indicate three lines of output with "a" followed by "b" followed by "c". If the statement causes an error, fill in the right-hand column with the phrase "error" to indicate this.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>var1.method1();</td>
<td></td>
</tr>
<tr>
<td>var1.method2();</td>
<td></td>
</tr>
<tr>
<td>var1.method3();</td>
<td></td>
</tr>
<tr>
<td>var2.method1();</td>
<td></td>
</tr>
<tr>
<td>var2.method2();</td>
<td></td>
</tr>
<tr>
<td>var2.method3();</td>
<td></td>
</tr>
<tr>
<td>var3.method1();</td>
<td></td>
</tr>
<tr>
<td>var4.method1();</td>
<td></td>
</tr>
<tr>
<td>var4.method2();</td>
<td></td>
</tr>
<tr>
<td>var4.method3();</td>
<td></td>
</tr>
<tr>
<td>((Don) var1).method2();</td>
<td></td>
</tr>
<tr>
<td>((Mike) var2).method2();</td>
<td></td>
</tr>
<tr>
<td>((Raph) var3).method1();</td>
<td></td>
</tr>
<tr>
<td>((Don) var3).method2();</td>
<td></td>
</tr>
<tr>
<td>((Leo) var4).method3();</td>
<td></td>
</tr>
</tbody>
</table>
2. Inheritance and Comparable Programming.
You have been asked to extend a pre-existing class Person that represents a person used as part of an online dating/marriage system. The Person class includes the following constructors and methods:

<table>
<thead>
<tr>
<th>Constructor/Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>public Person(String name)</td>
<td>constructs a person with the given name</td>
</tr>
<tr>
<td>public String getName()</td>
<td>returns the person's name</td>
</tr>
<tr>
<td>public void engageTo(Person partner)</td>
<td>sets the person to be engaged to the given partner</td>
</tr>
<tr>
<td>public Person getFiancee()</td>
<td>returns the person's fiancée, or null if none</td>
</tr>
<tr>
<td>public boolean isSingle()</td>
<td>returns true if the person has no fiancée</td>
</tr>
<tr>
<td>public Queue&lt;String&gt; getPreferences()</td>
<td>returns a queue of preferred partners</td>
</tr>
<tr>
<td>public Map&lt;String, Integer&gt; getRankings()</td>
<td>returns a map of rankings</td>
</tr>
<tr>
<td>public String toString()</td>
<td>returns a string representation of the person</td>
</tr>
</tbody>
</table>

You are to define a new class called Playa that extends this class through inheritance. A Playa should behave like a Person except that it makes mischief by allowing itself to be engaged to multiple persons at the same time, keeping track of a collection of all such fiancées. You should provide the same methods as the superclass, as well as the following new behavior:

<table>
<thead>
<tr>
<th>Constructor/Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>public Playa(String name)</td>
<td>constructs a playa with the given name</td>
</tr>
<tr>
<td>public int countFiances()</td>
<td>returns the number of fiancées to which this playa is currently engaged</td>
</tr>
</tbody>
</table>

The behaviors related to preferences and rankings of potential partners are unaffected by this subclass.

Some of the existing behaviors from Person should behave differently on Playa objects:

- When the engageTo method is called, the Playa should still retain the existing engageTo behavior (because it maintains important internal state), but it should also keep track of a collection of all engagement partners seen so far. Each partner passed to engageTo should become part of this collection. It should not be possible for the same person to appear twice in this collection. If null is passed to engageTo, your Playa should instead clear its collection of partners to become single again. (A Playa can become engaged to any person(s), not just other Playas.)

- The getFiancee method should return the partner to which the Playa most recently became engaged (not counting null). This will occur automatically if the original engageTo behavior from Person is retained.

- The isSingle method should return true only if the Playa has no partners in its engagement collection.

You must also make Playa objects comparable to each other using the Comparable interface. Playas are compared by their number of fiancées, breaking ties by name. In other words, a Playa object with fewer fiancées in its partner collection is considered to be "less than" one with more fiancées in its collection. If two Playa objects have the same number of fiancées, the one whose name comes first in alphabetical order is considered "less." If the two objects have the same number of fiancées and the same name, they are considered to be "equal."

The majority of your grade comes from implementing the correct behavior. Part of your grade also comes from appropriately utilizing the behavior you have inherited from the superclass and not re-implementing behavior that already works properly in the superclass.
3. **Linked List Programming.**

Write a method `expand` that could be added to the `LinkedIntList` class from lecture and section. The method accepts an integer `f` as a parameter and replaces every value `i` with `f` copies of the value `(i / f)`. Suppose a `LinkedIntList` variable `list` stores the following values:

\[ [21, 8, 15, 0, -3, 32] \]

The call `list.expand(3);` would change the list to store the following elements:

\[ [7, 7, 7, 2, 2, 2, 5, 5, 5, 0, 0, 0, -1, -1, -1, 10, 10, 10] \]

If an element of the original list is not evenly divisible by `f`, as with 8 and 32 above, the resulting list should truncate any fractional component (as is done naturally by integer division). If the parameter value passed is 1, the list is unchanged. If it is 0 or negative, the list should become empty.

For full credit, your solution must run in \(O(N)\) time where \(N\) is the length of the list. You may not call any methods of the linked list class to solve this problem, and you may not use any auxiliary data structure to solve this problem (such as an array, `ArrayList`, `Queue`, `String`, etc).

Assume that you are using the `LinkedIntList` and `ListNode` class as defined in lecture and section:

```java
public class LinkedIntList {
    private ListNode front;

    methods
}

public class ListNode {
    public int data;       // data stored in this node
    public ListNode next;  // a link to the next node in the list

    public ListNode() { ... }
    public ListNode(int data) { ... }
    public ListNode(int data, ListNode next) { ... }
}
```
4. **Searching and Sorting.**

(a) Suppose we are performing a **binary search** on a sorted array called `numbers` initialized as follows:

```java
int[] numbers = {-2, 0, 1, 7, 9, 16, 19, 28, 31, 40, 52, 68, 85, 99};
```

// search for the value 5
int index = binarySearch(numbers, 5);

Write the indexes of the elements that would be examined by the binary search (the mid values in our algorithm's code) and write the value that would be returned from the search. Assume that we are using the binary search algorithm shown in lecture and section.

- Indexes examined: __________________________________________________________
- Value Returned: __________________________________________________________

(b) Write the state of the elements of the array below after each of the first 3 passes of the outermost loop of the selection sort algorithm.

```java
int[] numbers = {63, 9, 45, 72, 27, 18, 54, 36};
selectionSort(numbers);
```

(c) Trace the complete execution of the merge sort algorithm when called on the array below, similarly to the example trace of merge sort shown in the lecture slides. Show the sub-arrays that are created by the algorithm and show the merging of sub-arrays into larger sorted arrays.

```java
int[] numbers = {63, 9, 45, 72, 27, 18, 54, 36};
mergeSort(numbers);
```
5. **Binary Search Trees.**

(a) Write the binary search tree that would result if these elements were added to an empty tree in this order:
- Meg, Stewie, Peter, Joe, Lois, Brian, Quagmire, Cleveland

(b) Write the elements of your tree above in the order they would be visited by each kind of traversal:

- **Pre-order:**

- **In-order:**

- **Post-order:**
6. **Binary Tree Programming.**

Write a method `nodesAtLevels` that could be added to the `IntTree` class from lecture and section. The method accepts minimum and maximum integers as parameters and returns a count of how many elements exist in the tree at those levels, inclusive. Recall that the root of a tree is at level 1, its children are at level 2, their children at level 3, and so on. The table below shows the results of several calls on an `IntTree` variable `tree`:

<table>
<thead>
<tr>
<th>Level</th>
<th>Call</th>
<th>Value Returned</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><code>tree.nodesAtLevels(2, 4)</code></td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td><code>tree.nodesAtLevels(4, 5)</code></td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td><code>tree.nodesAtLevels(1, 2)</code></td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td><code>tree.nodesAtLevels(3, 3)</code></td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td><code>tree.nodesAtLevels(7, 9)</code></td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td><code>tree.nodesAtLevels(5, 9)</code></td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td><code>tree.nodesAtLevels(1, 1)</code></td>
<td>1</td>
</tr>
</tbody>
</table>

For example, `tree.nodesAtLevels(4, 5)` returns 6 because -8, 80, -2, 57, 1, and 27 are in that range of levels.

Your method should throw an `IllegalArgumentException` if the minimum passed is less than 1 or is greater than the maximum passed. It is legal for the minimum and/or maximum to be larger than the height of the tree; a tree has 0 nodes at any levels that exceed its height. An empty tree contains 0 nodes at any level.

You may define private helper methods to solve this problem, but otherwise you may not call any other methods of the class nor create any data structures such as arrays, lists, etc. Your method should not change the structure or contents of the tree being examined.

Recall the `IntTree` and `IntTreeNode` classes as shown in lecture and section:

```java
class IntTreeNode {
    public int data;          // data stored in this node
    public IntTreeNode left;  // reference to left subtree
    public IntTreeNode right; // reference to right subtree

    IntTreeNode(int data) { ... }
    IntTreeNode(int data, IntTreeNode left, IntTreeNode right) { ... }
}

class IntTree {
    private IntTreeNode overallRoot;

    methods
    ...
}
```
7. **Binary Tree Programming.**
Write a method `trim` that could be added to the `IntTree` class from lecture and section. The method accepts minimum/maximum integers as parameters and removes from the tree any elements that are not in that range, inclusive. For this method, assume that your tree is a binary search tree (BST) and that its elements are in valid BST order. Your method should maintain the BST ordering property of the tree. For example, suppose a variable of type `IntTree` called `tree` stores the following elements:

```
      50
     /   \
    38    90
   /     /  \
  14     42  54
 /   \
20    72
```

The table below shows what the state of the tree would be if various `trim` calls were made. The calls shown are separate; it's not a chain of calls in a row. You may assume that the minimum is less than or equal to the maximum.

```
<table>
<thead>
<tr>
<th>tree.trim(25, 72);</th>
<th>tree.trim(54, 80);</th>
<th>tree.trim(18, 42);</th>
<th>tree.trim(-3, 6);</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>54</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>72</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>61</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>72</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>61</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>61</td>
<td>26</td>
<td></td>
</tr>
</tbody>
</table>
```

Hint: The BST ordering property is important for solving this problem. If a node's data value is too large or too small to fit within the range, this may also tell you something about whether that node's left or right subtree elements can be within the range. Taking advantage of such information makes it more feasible to remove the correct nodes.

You may define private helper methods to solve this problem, but otherwise you may not call any other methods of the class nor create any data structures such as arrays, lists, etc.