CSE 143

Lecture 9: introduction to recursion

reading: 12.1
Exercise

(To a student in the front row)
How many students total are directly behind you in your "column" of the classroom?

- You have poor vision, so you can see only the people right next to you. So you can't just look back and count.
- But you are allowed to ask questions of the person next to you.
- How can we solve this problem? *(recursively)*
The idea

• Recursion is all about breaking a big problem into smaller occurrences of that same problem.
  • Each person can solve a small part of the problem.
    • What is a small version of the problem that would be easy to answer?
    • What information from a neighbor might help me?
Recursive algorithm

- Number of people behind me:
  - If there is someone behind me, ask him/her how many people are behind him/her.
    - When they respond with a value $N$, then I will answer $N + 1$.
  - If there is nobody behind me, I will answer $0$. 
Recursion

- **recursion**: The definition of an operation in terms of itself.
  - Solving a problem using recursion depends on solving smaller occurrences of the same problem.

- **recursive programming**: Writing methods that call themselves to solve problems recursively.
  - An equally powerful substitute for *iteration* (loops)
  - Particularly well-suited to solving certain types of problems
a) Part of the Mandelbrot set.
b) Part of the North American coastline near Hudson Bay.
Linked Lists are Self-Similar

- a linked list is:
  - null
  - a node whose `next` field references a list

- **recursive data structure**: a data structure partially composed of smaller or simpler instances of the same data structure
Why learn recursion?

- "Cultural experience" – think differently about problems
- Solves some problems more naturally than iteration
- Can lead to elegant, simplistic, short code (when used well)
- Many programming languages ("functional" languages such as Scheme, ML, and Haskell) use recursion exclusively (no loops)
- A key component of many of our assignments in CSE 143
Recursion and cases

- Every recursive algorithm involves at least 2 cases:
  - **base case**: A simple occurrence that can be answered directly.
  - **recursive case**: A more complex occurrence of the problem that cannot be directly answered, but can instead be described in terms of smaller occurrences of the same problem.

- Some recursive algorithms have more than one base or recursive case, but all have at least one of each.
- A crucial part of recursive programming is identifying these cases.
Getting down stairs

- Need to know two things:
  - Getting down one stair
  - Recognizing the bottom

- Most code will look like:

```java
if (simplest case) {
    compute and return solution
} else {
    divide into similar subproblem(s)
    solve each subproblem recursively
    assemble the overall solution
}
```
Another recursive task

• How can we remove exactly half of the M&M's in a large bowl, without dumping them all out or being able to count them?
  
  • What if multiple people help out with solving the problem? Can each person do a small part of the work?

• What is a number of M&M's that it is easy to double, even if you can't count?
  
  • (What is a "base case"?)
Recursion in Java

- Consider the following method to print a line of * characters:

```java
// Prints a line containing the given number of stars.
// Precondition: n >= 0
public static void printStars(int n) {
    for (int i = 0; i < n; i++) {
        System.out.print("*");
    }
    System.out.println(); // end the line of output
}
```

- Write a recursive version of this method (that calls itself).
  - Solve the problem without using any loops.
  - Hint: Your solution should print just one star at a time.
A basic case

- What are the cases to consider?
  - What is a very easy number of stars to print without a loop?

```java
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else {
        ...
    }
}
```
Handling more cases

• Handling additional cases, with no loops (in a bad way):

```java
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else if (n == 2) {
        System.out.print("*");
        System.out.println("*");
    } else if (n == 3) {
        System.out.print("*");
        System.out.print("*");
        System.out.println("*");
    } else if (n == 4) {
        System.out.print("*");
        System.out.print("*");
        System.out.print("*");
        System.out.println("*");
    } else ...
}
```
Handling more cases 2

- Taking advantage of the repeated pattern (somewhat better):

```java
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else if (n == 2) {
        System.out.print("*");
        printStars(1); // prints "*
    } else if (n == 3) {
        System.out.print("*");
        printStars(2); // prints "**"
    } else if (n == 4) {
        System.out.print("*");
        printStars(3); // prints "***"
    } else ...
}
```
Using recursion properly

- Condensing the recursive cases into a single case:

```java
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else {
        // recursive case; print one more star
        System.out.print("*");
        printStars(n - 1);
    }
}
```
"Recursion Zen"

- The real, even simpler, base case is an `n` of 0, not 1:

```java
public static void printStars(int n) {
    if (n == 0) {
        // base case; just end the line of output
        System.out.println();
    } else {
        // recursive case; print one more star
        System.out.print("*");
        printStars(n - 1);
    }
}
```

- **Recursion Zen**: The art of properly identifying the best set of cases for a recursive algorithm and expressing them elegantly.

  (A CSE 143 informal term)
Recursive tracing

Consider the following recursive method:

```java
public static int mystery(int n) {
    if (n < 10) {
        return n;
    } else {
        int a = n / 10;
        int b = n % 10;
        return mystery(a + b);
    }
}
```

What is the result of the following call?
mystery(648)
A recursive trace

mystery(648):
- int a = 648 / 10;  // 64
- int b = 648 % 10;  // 8
- return mystery(a + b);  // mystery(72)

mystery(72):
- int a = 72 / 10;  // 7
- int b = 72 % 10;  // 2
- return mystery(a + b);  // mystery(9)

mystery(9):
- return 9;
Recursive tracing 2

- Consider the following recursive method:

```java
public static int mystery(int n) {
    if (n < 10) {
        return (10 * n) + n;
    } else {
        int a = mystery(n / 10);
        int b = mystery(n % 10);
        return (100 * a) + b;
    }
}
```

- What is the result of the following call?
  
  `mystery(348)`
A recursive trace 2

mystery(348)

- int a = mystery(34);
  - int a = mystery(3);
    - return (10 * 3) + 3; // 33
    - int b = mystery(4);
      - return (10 * 4) + 4; // 44
      - return (100 * 33) + 44; // 3344
  - int b = mystery(8);
    - return (10 * 8) + 8; // 88
    - return (100 * 3344) + 88; // 334488

- What is this method really doing?
Exercise

• Write a recursive method `reverseLines` that accepts a file `Scanner` and prints the lines of the file in reverse order.

  • Example input file:
    ```
    I have eaten
    the plums
    that were in
    the icebox
    ```

  • Expected console output:
    ```
    the icebox
    that were in
    the plums
    I have eaten
    ```

  • What are the cases to consider?
    • How can we solve a small part of the problem at a time?
    • What is a file that is very easy to reverse?
Reversal pseudocode

- Reversing the lines of a file:
  - Read a line L from the file.
  - Print the rest of the lines in reverse order.
  - Print the line L.

- If only we had a way to reverse the rest of the lines of the file....
Reversal solution

```java
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        // recursive case
        String line = input.nextLine();
        reverseLines(input);
        System.out.println(line);
    }
}
```

- Where is the base case?
## Tracing our algorithm

- **call stack:** The method invocations currently running

```
reverseLines(new Scanner("poem.txt"));
```

```java
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        String line = input.nextLine(); // "I have eaten"

        reverseLines(input);
        System.out.println(line);
    }
}
```

Output:

```
I have eaten
the plums
that were in
the icebox
```