Lecture 21: Binary Search Trees; TreeSet

For added security, after we encrypt the data stream, we send it through our Navajo code talker.

...is he just using Navajo words for "zero" and "one"?

Whoa, hey, keep your voice down!
Recall: \( x = \text{change}(x) \)

- Methods that modify a tree should have the following pattern:
  - input (parameter): old state of the node
  - output (return): new state of the node

- In order to actually change the tree, you must reassign:

  ```
  node = \text{change}(node, \text{parameters});
  node.left = \text{change}(node.left, \text{parameters});
  node.right = \text{change}(node.right, \text{parameters});
  overallRoot = \text{change}(overallRoot, \text{parameters});
  ```
Exercise

- Add a method `getMin` to the `IntTree` class that returns the minimum integer value from the tree. Assume that the elements of the `IntTree` constitute a legal binary search tree. Throw a `NoSuchElementException` if the tree is empty.

```java
int min = tree.getMin(); // -3
```
Exercise solution

// Returns the minimum value from this BST.
// Throws a NoSuchElementException if the tree is empty.
public int getMin() {
    if (overallRoot == null) {
        throw new NoSuchElementException();
    }
    return getMin(overallRoot);
}

private int getMin(IntTreeNode root) {
    if (root.left == null) {
        return root.data;
    } else {
        return getMin(root.left);
    }
}
Exercise

• Add a method `remove` to the `IntTree` class that removes a given integer value from the tree, if present. Remove the value in such a way as to maintain BST ordering.

- `tree.remove(73);`
- `tree.remove(29);`
- `tree.remove(87);`
- `tree.remove(55);`
Cases for removal 1

1. a leaf:

2. a node with a left child only:

3. a node with a right child only:

replace with null
replace with left child
replace with right child

```
overall root

55
29
-3 42

overall root

55
29
42

overall root

29
42

overall root

42

```

tree.remove(-3);
tree.remove(55);
tree.remove(29);

tree.remove(29);
Cases for removal 2

4. a node with both children: replace with \textbf{min from right}
   - (replacing with max from left would also work)

```
overall root
55
   29
   87
   -3 42 60 91

overall root
60
   29
   87
   -3 42 91

tree.remove(55);
```
Exercise solution

// Removes the given value from this BST, if it exists.
public void remove(int value) {
    overallRoot = remove(overallRoot, value);
}

private IntTreeNode remove(IntTreeNode root, int value) {
    if (root == null) {
        return null;
    } else if (root.data > value) {
        root.left = remove(root.left, value);
    } else if (root.data < value) {
        root.right = remove(root.right, value);
    } else {
        // root.data == value; remove this node
        if (root.right == null) {
            return root.left;  // no R child; replace w/ L
        } else if (root.left == null) {
            return root.right;  // no L child; replace w/ R
        } else {
            // both children; replace w/ min from R
            root.data = getMin(root.right);
            root.right = remove(root.right, root.data);
        }
    }
    return root;
}
Searching BSTs

- The BSTs below contain the same elements.
  - What orders are "better" for searching?
Trees and balance

- **balanced tree**: One whose subtrees differ in height by at most 1 and are themselves balanced.
  - A balanced tree of N nodes has a height of $\sim \log_2 N$.
  - A very unbalanced tree can have a height close to N.

- The runtime of adding to / searching a BST is closely related to height.

- Some tree collections (e.g. TreeSet) contain code to balance themselves as new nodes are added.