CSE 143

Lecture 19: Binary Trees

reading: 17.1 – 17.3
Trees

- **tree**: A directed, acyclic structure of linked nodes.
  - *directed*: Has one-way links between nodes.
  - *acyclic*: No path wraps back around to the same node twice.

- **binary tree**: One where each node has at most two children.

- **Recursive definition**: A tree is either:
  - empty (*null*), or
  - a *root* node that contains:
    - *data*,
    - a *left* subtree, and
    - a *right* subtree.
      - (The left and/or right subtree could be empty.)
Trees in computer science

- TreeMap and TreeSet implementations
- folders/files on a computer
- family genealogy; organizational charts
- AI: decision trees
- compilers: parse tree
  - $a = (b + c) * d$;
- cell phone T9

![Diagram of a parse tree for the expression $a = (b + c) * d$]

![Diagram of a family tree with names: Joe, John, Jane, Jack]
Terminology

- **node**: an object containing a data value and left/right children
  - **root**: topmost node of a tree
  - **leaf**: a node that has no children
  - **branch**: any internal node; neither the root nor a leaf
  - **parent**: a node that refers to this one
  - **child**: a node that this node refers to
  - **sibling**: a node with a common parent

- **subtree**: the smaller tree of nodes on the left or right of the current node
- **height**: length of the longest path from the root to any node
- **level** or **depth**: length of the path from a root to a given node
Recursive definition: A tree is either:
- empty (null), or
- a root node that contains:
  - data,
  - a left tree, and
  - a right tree
A tree node for integers

- A basic **tree node object** stores data, refers to left/right
- Multiple nodes can be linked together into a larger tree

![Tree Diagram]

- 42
- 59
- 27
- 86
// An IntTreeNode object is one node in a binary tree of ints.
public class IntTreeNode {
    public int data; // data stored at this node
    public IntTreeNode left; // reference to left subtree
    public IntTreeNode right; // reference to right subtree

    // Constructs a leaf node with the given data.
    public IntTreeNode(int data) {
        this(data, null, null);
    }

    // Constructs a branch node with the given data and links.
    public IntTreeNode(int data, IntTreeNode left, IntTreeNode right) {
        this.data = data;
        this.left = left;
        this.right = right;
    }
}
// An IntTree object represents an entire binary tree of ints.
public class IntTree {
    private IntTreeNode overallRoot;  // null for an empty tree

    methods
}

- Client code talks to the IntTree, not to the node objects inside it.
- Methods of the IntTree create and manipulate the nodes, their data and links between them.
IntTree constructors

• For now, assume we have the following constructors:

public IntTree(IntTreeNode overallRoot)
public IntTree(int height)

- The 2nd constructor will create a tree and fill it with nodes with random data values from 1-100 until it is full at the given height.

IntTree tree = new IntTree(3);
Traversals

- **traversal**: An examination of the elements of a tree.
  - A pattern used in many tree algorithms and methods

- Common orderings for traversals:
  - **pre-order**: process root node, then its left/right subtrees
  - **in-order**: process left subtree, then root node, then right
  - **post-order**: process left/right subtrees, then root node
Traversal example

- **pre-order:** 17 41 29 6 9 81 40
- **in-order:** 29 41 6 17 81 9 40
- **post-order:** 29 6 41 81 40 9 17
Traversal trick

- To quickly generate a traversal:
  - Trace a path around the tree.
  - As you pass a node on the proper side, process it.
    - pre-order: left side
    - in-order: bottom
    - post-order: right side

- pre-order: 17 41 29 6 9 81 40
- in-order: 29 41 6 17 81 9 40
- post-order: 29 6 41 81 40 9 17
Exercise

- Give pre-, in-, and post-order traversals for the following tree:

  - pre: 42 15 27 48 9 86 12 5 3 39
  - in: 15 48 27 42 86 5 12 9 3 39
  - post: 48 27 15 5 12 86 39 3 42
Exercise

• Add a method `print` to the `IntTree` class that prints the elements of the tree, separated by spaces.

  – A node's left subtree should be printed before it, and its right subtree should be printed after it.

  – Example: `tree.print();`

```plaintext
29 41 6 17 81 9 40
```
Exercise solution

// An IntTree object represents an entire binary tree of ints.
public class IntTree {
    private IntTreeNode overallRoot;  // null for an empty tree ...

    public void print() {
        print(overallRoot);
        System.out.println();  // end the line of output
    }

    private void print(IntTreeNode root) {
        // (base case is implicitly to do nothing on null)
        if (root != null) {
            // recursive case: print left, center, right
            print(overallRoot.left);
            System.out.print(overallRoot.data + " ");
            print(overallRoot.right);
        }
    }
}
Template for tree methods

```java
global class IntTree {
private IntTreeNode overallRoot;
...

global type name(parameters) {
    name(overallRoot, parameters);
}

global type name(IntTreeNode root, parameters) {
    ...
}
}
```

- Tree methods are often implemented recursively
  - with a public/private pair
  - the private version accepts the root node to process
Exercise

• Add a method contains to the IntTree class that searches the tree for a given integer, returning true if it is found.

  – If an IntTree variable tree referred to the tree below, the following calls would have these results:

    • tree.contains(87) → true
    • tree.contains(60) → true
    • tree.contains(63) → false
    • tree.contains(42) → false
// Returns whether this tree contains the given integer.
public boolean contains(int value) {
    return contains(overallRoot, value);
}

private boolean contains(IntTreeNode node, int value) {
    if (node == null) {
        return false; // base case: not found here
    } else if (node.data == value) {
        return true; // base case: found here
    } else {
        // recursive case: search left/right subtrees
        return contains(node.left, value) ||
               contains(node.right, value);
    }
}
Exercise

• Add a method named `printSideways` to the `IntTree` class that prints the tree in a sideways indented format, with right nodes above roots above left nodes, with each level 4 spaces more indented than the one above it.

  – Example: Output from the tree below:
Exercise solution

// Prints the tree in a sideways indented format.
public void printSideways() {
    printSideways(overallRoot, "")
}

private void printSideways(IntTreeNode root, String indent) {
    if (root != null) {
        printSideways(root.right, indent + "    ");
        System.out.println(indent + root.data);
        printSideways(root.left, indent + "    ");
    }
}