

Lecture 10: Intro to Recursion

- This week we're taking a break from data structures --> a new CONTROL STRUCTURE
 - We've been writing lots of code with LOOPS
 - This technique is called ITERATION
 - Now we're going to look at an alternative technique called RECURSION
 - Easy to explain, but hard to understand
 - You'll have an "AH HA!" moment
 - Equally powerful as loops
 - Why study if we can't get anything extra out of it?
 - Recursion vs. iteration is a fundamental debate in computer science (almost religious - a la Christianity vs Buddhism)
 - You need to be aware of this debate
 - One is not right or wrong - they are attempting to solve the same problem differently
 - Some programming languages ONLY have recursion - no loops
 - Also, some problems are naturally easier to express recursively vs. iteratively
- A non-programming example
 - "What row are you in?"
 - The iterative solution is to have the student count
 - Start at the front of the room, and do "counter++", moving eyes back by one row each time, while there are still rows to count
 - But now imagine instead of people, you are robots and you have very poor vision
 - You can't count all the way to the front of the room
 - You can only see/communicate with the robots next to you
 - What can you ask the robot in front of you that would help?
 - NOT "Do you know what row I'm in?"
 - "What row are YOU in?"
 - Then the robot in front can ask the person in front of it, etc.
 - Until when?
 - The first row - the robot can see that no one is in front of it
 - Each robot then responds with the row # plus 1
 - This is RECURSION
 - Break a problem into smaller pieces that are somehow all the same
 - Defining an operation in terms of itself
- Also, consider walking down a staircase (draw picture)
 - You only have to know how to do two things:
 - (1) how to take a step downwards
 - (2) how to recognize that you're at the bottom
 - Then you can put the steps together
 - We distinguish between these two cases:
 - The BASE CASE: when to stop
 - The RECURSIVE CASE: how to take a step to a smaller problem (taking a step down decreases the total number of steps left)

- If we didn't have the BASE CASE...
 - You'd "trip and fall" - something bad
- If we didn't have the RECURSIVE CASE...
 - You'd never get down the stairs at all
- Some problems may have more than one base case (ways to stop) or more than one recursive case (ways to step down), but all problems will have at least 1 of each
- Another example
 - I have a bowl of M&Ms
 - I want to double the number of M&Ms in the bowl
 - But I can't count them
- Programming example: writeStars(int n)
 - Produces a line of output with n stars (for some $n \geq 0$)
 - (show the iterative solution)
 - Initialization before the loop (*int i = 0*)
 - A test at each iteration of the loop ($i < n$)
 - Some code run at each iteration (*print* and $i++$)
 - Concluding code (*println*)
 - We want to write this recursively, so we think of CASES
 - Recursion always involves case analysis
 - Usually I try to think of the base case first - always ask "What is my base case?"
 - But for you, the recursive case may be easier to do first -
 - Base case? What is the EASIEST line of stars to write?
 - One star?
 - Actually, zero stars is even easier!
 - We just print a blank line
 - This forms our base case:


```

              if (n == 0) {
                  System.out.println();
              } else {
                  ...
              }
              
```
 - If-else statements are common in recursion - either we have reached the end (stopping condition - base case), or we have some work left to do
 - Recursive case? (this can be an "ah ha!" solution - so don't blurt it out!)
 - What you'd want to do is use a for-loop to print all the stars...
 - But we can't do that. Instead we have to do a piece of the task and then delegate
 - What small piece of the task can we do? Get a little closer to being done?
 - Print a single star
 - Now what is left to do?
 - Print $n - 1$ stars
 - How can we do that?

- This is what we call the LEAP OF FAITH
 - “If only we had a method that could write n-1 stars, we could call it”
 - But we do have that method - the one we’re writing!

```
public void writeStars2(int n) {
    if (n == 0)
        System.out.println();
    else {
        System.out.print("*");
        writeStars2(n - 1);
    }
}
```

- This seems wrong to us b/c we’re used to solving the entire problem ourselves
- How can we call the same method again and it will work?
- Don’t think of calling the same method again - it’s another robot with the same code
- Also, *n* is smaller - what would happen if we called writeStars2(*n*) again?
- Explain how this relates to the robots-counting-row-number example

```
writeStars2(2)
    writeStars2(1)
        writeStars2(0)
```

- Another example: print lines from a file in reverse order
 - How would we do this with a loop?
 - Remember, we need to print the last line first! So we need to “remember” the first lines
 - So we’d need a structure - like an ArrayList - to store the lines
 - But with recursion, we don’t need an extra structure
 - Write the header - take a Scanner as a parameter
 - What is the easiest file to reverse?
 - A 1-line file? Actually, a 0-line file
 - What do we want to do in this “base case”? How do reverse 0 lines?

- You do nothing!

```
if (input.hasNextLine()) {
}
```

- We could leave a blank if-statement, but it’s usually better practice in this case to reverse the condition and have a blank else - and then just leave off the else altogether! (else is “implied”)

```
if (input.hasNextLine()) {
    ...
}
```

- OK, now the recursive case
 - We need to do a little bit of work - how do we do that?
 - Read a line of the file
 - “this”
 - What work is left to do after that?
 - “is/fun/no?”
 - What happens if we make a recursive call?
 - LEAP OF FAITH that the recursive call will reverse “is/fun/no?” into “no?/fun/is”
 - What is the result that we want?
 - “no?/fun/is/this”
 - So if we make a recursive call, what work is left to do?
 - Just print the final line
- So we can complete the code!
- Wow, can this be it?
- Simulate a call on “reverse” using paper (below)
 - Visual representation of what is going on
 - We call the stack of papers the “call stack”
- Another example: stutter an integer
 - Ex. given 348, turn it into 334488
 - Replace each digit with 2 of that digit
 - Base case: what is the easiest number to stutter?
 - 0 - incorrect! We can’t really return “00” because that’s not a valid integer
 - It turns out that we’ll just return 0 in that case
 - So what’s an alternative easy number to stutter?
 - Any number < 10 (one digit)
 - (note: you can’t have 0 digits!)
 - How do we stutter a digit (for the base case)?
 - well, 1 --> 11, 2 --> 22, 3 --> 33,...
 - $n0 + n = nn$ (10 of $n + n$), or $11n$
 - so we can do $11*n$
 - So how do we deal with larger numbers - how can we split them up?
 - We need to split into smaller chunks
 - Since our base case is 1 digit, we need to process 1 digit
 - We can do this with $/10$ and $\%10$
 - Then we have 34 and 8. Let’s trust that our method works; we could call it to turn these into 3344 and 88.
 - How do we combine 3344 and 88 into a single number?
 - Can’t simple add --> would give 3432
 - Instead, we want 334 THOUSAND - we need to shift over the 3344!
 - Multiply by 100
 - What about negative numbers?
 - Special case for these

(if time, writeBinary) - same trick but /2 and %2 - can you tell anything about the binary from just the number? You can tell what it ends with

```

+-----+
| public void reverse(Scanner input) { |
|     if (input.hasNextLine()) { |
|         String line = input.nextLine(); |
|     --> reverse(input); |
|         System.out.println(line); |
|     } |
| } |
|     +-----+ |
| line | "this" | |
|     +-----+ |
+-----+
+-----+
| public void reverse(Scanner input) { |
| +-----+ |
| | public void reverse(Scanner input) { |
| |     if (input.hasNextLine()) { |
| |         String line = input.nextLine(); |
| |     --> reverse(input); |
| |         System.out.println(line); |
| |     } |
| | } |
| |     +-----+ |
+--| line | "is" | |
|     +-----+ |
| +-----+ |
+-----+
| public void reverse(Scanner input) { |
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| | |         String line = input.nextLine(); |
| | |     --> reverse(input); |
| | |         System.out.println(line); |
| | |     } |
+--| | } |
| |     +-----+ |
+--| line | "fun" | |
|     +-----+ |
| +-----+ |
+-----+

```

```

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| | |     } |
+--| | } |
| |     +-----+ |
+--| line | "no?" |
| |     +-----+ |
+-----+

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| | | |     +-----+ |
+--| line | | |
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|     +-----+ |
| line | "this" |
|     +-----+ |
+-----+

```

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